SAMSA 2022 Maputo

60 ANOS DE ENSINO SUPERIOR 1962 - 2022

Southern Africa Mathematical Sciences Association Annual Conference

Sciences for Technology Innovation and Human Development

22-25 November, 2022 / UEM Main Campus, Maputo

BOOK OF ABSTRACTS

Organizers

Sponsors

Contents

ABSTRACTS

1 PLENARY SPEAKERS

Modelling, analysis and simulations of bulk-surface PDEs with applications to experimental sciences

Anotida Madzvamuse
University of British Columbia & University of Sussex
A.Madzvamuse@sussex.ac.uk

Abstract

In this talk, I will present bulk-surface PDEs, were we couple partial differential equations of reaction-diffusion type in the bulk of the domain to reaction-diffusion systems on the surface of the domain. These two systems are then coupled through appropriate boundary conditions. Under appropriate assumptions, I will then present some analytical results which are subsequently validated through numerical simulations. In particular, it turns out that pattern formation for bulk-surface reaction-diffusion systems is richer than that obtained with classical models on uncoupled domains. Finally, I will conclude by presenting a bulk-surface wave pinning model in the area of cell motility.

****

Neural field models with and without microstructure

Arkadi Ponossov[a], **Evgenii Burlakov**[b], **John Wyller**[a] & **Evgeny Zhukovsky**[c]
[a] Norwegian University of Life Sciences, 1432 Ås, Norway
arkadi@nmbu.no, john.wyller@nmbu.no

[b] X-Bio Institute, University of Tyumen, 625003 Tyumen, Russia
eb_@bk.ru

[c] Tambov State University, 392000 Tambov, Russia
zukovskys@mail.ru

Abstract

We consider a class of non-local partial differential equations covering several important neural field models, including the Wilson-Cowan model, the Amari model, the Hopfield network and models with periodic and aperiodic microstructure. We suggest conditions for existence and uniqueness of solutions and study the property of continuous dependence of these solutions on the spatiotemporal integration kernel, delays, external inputs and measures describing microstructure, with a special emphasis on the steepness parameter, which describes the neurons'

activation rate. Finally, we establish a connection between the Wilson-Cowan and Hopfield network models with and without delays. The theories of local Volterra contractions and fixed-point indices are used to prove these results.

Keywords: Neural field equations, well-posedness, stationary and travelling wave solutions, sigmoid and Heaviside functions, weak convergence of measures

Category: **Analysis, Mathematical Biology**

****

Is there a possible synergy between HIV and COVID-19?

E Lungu
Department of Mathematics and Statistical Sciences, Botswana International University of Science and Technology, Private Bag 16, Palapye, Botswana
lungue@biust.ac.bw

Abstract

Humans have been exposed to infectious viral diseases from the beginning of time. Perhaps the most common human viral infections are the strains of the common cold (flu). Some of the flu strains are very virulent and can cause fatalities, especially among the elderly.

Viruses continuously mutate, a characteristic which makes it difficult to develop vaccines that protect against emerging strains. We begin by giving an overview of viral infections for which lifelong vaccine protection has been achieved.

The measles vaccine (MMR) introduced in 1963 is administered as two doses, the first at 12-15 months of age and the second at 4-6 years old. Nearly all those who do not develop immunity after the first dose develop it after a second dose. The MMR vaccine also protects against Mumps and Rubella, and the immunity lasts a lifetime.

Chickenpox, a viral infection with the Varicella Zoster virus, caused so much suffering among children before the Varicella vaccine was developed and made commercially available in 1984. The Varicella vaccine is administered as two doses. The first dose prevents 95% of moderate disease and 100% of severe disease. Before the human immunodeficiency syndrome (HIV) appeared as a human infection, chickenpox infected an individual only once during their lifetime.

Shingles or Herpes Zoster is caused by the Varicella Zoster, the same virus that causes chickenpox. Shingles can develop only after initial infection with chickenpox or rarely after vaccination for chickenpox. After a person recovers from Shingles, the virus remains latent in the body, but many years after infection the virus can be reactivated (usually by HIV) but the individual will develop Chickenpox not Shingles. Shingles infection is the first sign of HIV infection.

Although there is no direct link suggesting a causal relationship between Ebola and HIV, there is strong evidence suggesting that antiretroviral therapy is beneficial to the prognosis of Ebola in individuals co-infected with Ebola and HIV. What is not disputable is the fact that the number of Ebola outbreaks has increased from one outbreak in eight years between 1976 and 1995 to

annual outbreaks after 1995, a period which coincides with peak infections for HIV. Ebola vaccines are at a developmental state with only one vaccine, rVSV-ZEBOV, approved in December 2019 in the United States.

SARS-COV-2 (COVID-19), identified in Wuhan (China) in 2019, is the seventh human coronavirus. While other human coronaviruses were quickly contained by various measures such as lockdowns, wearing of masks e.t.c, the same measures did not have the same desired effect on slowing the number of COVID-19 infections. So far over 600 million individuals have been infected and over 6 million deceased. Between 2020 and 2021 several vaccines have been developed. However, these vaccines lose their efficacy so fast that some individuals have already taken two doses (as required) and two boosters. This is unlike the chickenpox and measles vaccines.

This overview looks at the possible synergy in HIV and COVID-19 co-infection. We have constructed a mathematical model to investigate the influence on human immunity caused by the co-infection. We want to quantify the conditions under which infection with COVID-19 would worsen the prognosis of HIV or vice versa? Important deductions are made from the co-morbidity reproduction number, especially regarding the efficacy of HIV drugs.

Keywords: Malaria, Fake drugs, Hierachy of resort, Probability of extinction

Category: **Mathematical Biology**

****

Rainbow Spanning Trees in Edge-Colored Complete Graphs

Katherine Perry[a]
[a] Soka University of America
kperry@soka.edu

Abstract

A spanning tree of an edge-colored graph is rainbow provided that each of its edges receives a distinct color. In 1996, Brualdi and Hollingsworth conjectured that if K_{2m} is properly $(2m-1)$-edge-colored, then the edges of K_{2m} can be partitioned into m rainbow spanning trees, except when $m = 2$. In this talk, we'll look at the history and recent results concerning this conjecture and related questions and also consider the extremal question of maximizing and minimizing the number of rainbow spanning trees in K_n, given a special type of $(n-1)$-edge-coloring which is surjective and rainbow cycle free, called a *JL-coloring*.

Keywords: rainbow spanning tree, JL-coloring, complete graph

Category: **Graph Theory**

****

Gender and mathematics: status and sociocultural challenges

Marcos Cherinda
UNESCO Maputo Office

Abstract

Gerdes (2000) states that one of the characteristics of the history of mathematics is the dialectical interaction between extra-mathematicaland intra-mathematical factors stimulating (or holding back) the development of mathematical ideas. By intra-mathematical factors is meant, for example, the reflection of a mathematician on a certain mathematical problem already known in the literature. By extra-mathematical factors is meant, for example, the discovery of the concept of a regular hexagon and some of its properties in the context of searching for solutions to produce a fishing trap. In the session, I will present some examples of Gerdes (1952-2014) reasoning experience that show the process of moving from an extra-mathematical factor to intra-mathematical reasoning, illustrating the potentials ofethnomathematics in its anthropological, historical, philosophical and educational relevance.

****

Gender and mathematics: status and sociocultural challenges

Sarifa Abdul Magide Fagilde
Faculty of Natural Science, Mathematics and Statistics, Rovuma University
samfagilde@hotmail.com

Abstract

Gender differences in Mathematics performance and post compulsory mathematics courses, have been attracting the researcher's attention, in the last decades, and claiming for action to reverse the position of Mathematics being seen as a difficult subject and particularly as a male domain. Considering that, in one side, cultural paradigms govern the people's mind, influences decision making and, on the other side, in order to better understand the sociocultural challenges, where relations between genders is a concern, there are cultural dualisms that have been sectorizing activities and people and such sectorizing dualism, in the historical process, determined the professional orientation of generations, it is important to get to know the women status, this communication aims to address aspects related to women status and sociocultural beliefs in relation to women and mathematics and highlights possible interventions in order to bring a contribution, to reverse the current mind situation.

Keywords: Gender, mathematics, cultural dualism, challenges

Category: **Mathematics Education**

****

2 ALGEBRA

On the Fischer-Clifford matrices and character table of a group of shape $p^{1+2n}.G$

Abraham Love Prins
Nelson Mandela University, South Africa
abraham.prins@mandela.ac.za/abrahamprinsie@yahoo.com

Abstract

Let $\overline{G} = P.G$ be a finite extension where P is an extra-special p-group p^{1+2n}. The center $Z(P)$ of P is a cyclic group $\mathbb{Z}_p$ of order p and since $Z(P)$ is a characteristic subgroup of P, it follows that $Z(P)$ is normal in $\overline{G}$. Hence we obtain the factor group $\overline{F} = \frac{\overline{G}}{Z(P)} \cong p^{2n}.G$ which is an extension of an elementary abelian p-group p^{2n} by a finite group G. Moreover, the ordinary irreducible characters Irr$(\overline{F})$ of $\overline{F}$ can be lifted to $\overline{G}$, where the set Irr$(\overline{F})$ is identified with $\chi_i \in$ Irr$(\overline{G})$ such that $Z(P) \leq$ Ker(χ_i). In [1], a powerful method is described to construct the ordinary character table of an extension group $P.G$ by means of so-called Fischer-Clifford matrices $M(g_i)$ which are associated with the conjugacy classes $[g_i]$ of G. Since the irreducible characters of the factor group $\overline{F}$ can be lifted to $\overline{G}$, it follows that a Fischer-Clifford matrix $\widehat{M(g)}$ of the factor group $\overline{F}$ is embedded in the corresponding Fischer-Clifford matrix $M(g)$ of $\overline{G}$. Hence, analogous to the above lifting process, the Fischer-Clifford matrices $\widehat{M(g_i)}$ of $\overline{F}$ can be "lifted" to the matrices $M(g_i)$ in $\overline{G}$. Having obtained the matrices $M(g_i)$ from the matrices $\widehat{M(g_i)}$, the ordinary irreducible character table of $\overline{G}$ can be constructed using the Fischer-Clifford matrices technique as discussed in, for example, [2]. The approach described above works very well on an extension group, where the kernel is an extra-special p-group p^{1+2n}. In this paper, the author will elaborate more about this "lifting of Fischer-Clifford matrices" approach and illustrate it with an appropriate example.

Keywords: Fischer-Clifford matrices, inertia factors, character table, extension group

Category: **Algebra**

References

[1] B. Fischer, *Clifford-matrices*, Progr. Math. **95**, Michler G.O. and Ringel C.(eds), Birkhauser, Basel (1991), 1 - 16.

[2] A.L. Prins, *A maximal subgroup* 2^{4+6}:$(A_5 \times 3)$ *of* $G_2(4)$ *treated as a non-split extension* $\overline{G} = 2^{6\cdot}(2^4$:$(A_5 \times 3))$, Advances in Group Theory and Applications, **10** (2020), 43-66.

****

On products of supersoluble subgroups

Adolfo Ballester-Bolinches[a], **Sesuai Madanha**[b], **Tendai Mudziiri Shumba**[c] & **Maria Pedraza-Aguilera**[d]
[a] Universitat de València
[b] University of Pretoria
sesuai.madanha@up.ac.za
[c] Sobolev Institute of Mathematics
[d] Universitat Politècnica de València

Dedicated to the memory of Alexander Grant Robertson Stewart

Abstract

In 1957, Reinhold Baer proved that if G is the product of two normal supersolvable subgroups and G' is nilpotent, then G is supersolvable. There has been several generalisations of this result. In this project the structure of finite groups $G = AB$ which are a weakly mutually sn-permutable product of the subgroups A and B, that is, such that A permutes with every subnormal subgroup of B containing $A \cap B$ and B permutes with every subnormal subgroup of A containing $A \cap B$, is studied. We generalise Baer's theorem.

Keywords: supersoluble groups, mutually permutable products, normal products, derived subgroup

Category: **Algebra**

****

Applications of reduced and coreduced modules

David Ssevviiri
Department of Mathematics, Makerere University, Kampala, Uganda
Email: david.ssevviiri@mak.ac.ug

Abstract

Let R be a commutative unital ring and I an ideal of R. We show that I-reduced R-modules and I-coreduced R-modules provide a setting in which the Matlis-Greenless-May (MGM) Equivalence and the Greenless-May (GM) Duality hold. These two notions have been hitherto known to exist in the derived category setting. We realise the I-torsion and the I-adic completion functors as representable functors and under suitable conditions compute natural transformations between them and other functors.

Keywords: Reduced and coreduced modules, torsion and adic completion.

Category: **Algebra**

References

[1] L. Alonso Tarrio, A. Jeremias Lopez and J. Lipman, Local homology and cohomology on schemes, *Annales Scientifiques de l Ecole Normale Supérieure*, (4), **30**(1), (1997), 1–39.

[2] I. Assem, D. Simson, and A. Skowronski, "Elements of the Representation Theory of Associative Algebras: Volume 1: Techniques of Representation Theory", Cambridge University Press, 2006.

[3] M. Auslander, M. I. Platzeck and G. Todorov, Homological theory of idempotent ideals. *Trans. Amer. Math. Soc.* **332**(2), (1992), 667–692.

[4] T. Barthel, D. Heard and G. Valenzuela, Local duality in algebra and topology, *Adv. Math.* **335**, (2018), 563-663.

****

Representations of polynomial covariance commutation relations by linear operators in L_p spaces

Domingos Djinja[a], **Sergei Silvestrov**[b] & **Alex Tumwesegye**[c]

[a] Department of Mathematics and Informatics, Faculty of Sciences, Eduardo Mondlane University, Box 257, Maputo and Division of Applied Mathematics and Physics, UKK, Malardalen University, Box 883, Vasteras
domingos.djindja@uem.ac.mz/domingos.celso.djinja@mdu.se

[b] Division of Applied Mathematics and Physics, UKK, Malardalen University, Box 883, Vasteras
sergei.silvestrov@mdu.se

[c] Department of Mathematics, College of Natural Sciences, Makerere University, Box 7062, Kampala
alex.tumwesigye@mak.ac.ug

Abstract

In this work, representations of polynomial covariance type commutation relations are constructed. Representations involve pairs of linear integral and multiplication operator in L_p spaces. Representations theorems of Linear functional in L_p spaces are used to construct representations of commutations relations. Examples for specific and known algebras such as Heisenberg-Lie type, for instance, are presented.

Keywords: Commutation relations, representations, integral operators, multiplication operator

Category: **Algebra**

References

[1] OXTOBY, J. C. (1971): *Measure and category*. Springer-Verlag, New York.

[2] PERSSON, T. AND SILVESTROV, S. D. (2002): Commuting operators for representations of commutation relations defined by dynamical systems. *Numerical Functional Analysis and Optimization*, **33**: 7-9, 1146-1165.

[3] RUDIN, W. (1987): *Real and Complex Analysis*. 3rd ed, Mc Graw Hill.

[4] SAMOILENKO Y. S. (1991): *Spectral Theory of families of self-adjoint operators.* Kluwer Academic Publ.

****

Hochschild cohomology of Sullivan algebras and mapping spaces between manifolds

Jean Baptiste Gatsinzi[a]
[a] Botswana INternational University of Science and Technology
gatsinzij@biust.ac.bw

Abstract

Let $e : N^n \to M^m$ be an embedding of closed, oriented manifolds of dimension n and m respectively. We study the relationship between the homology of the free loop space LM on M and of the space $L_N M$ of loops of M based in N and define a shriek map $H_*(e)_! : H_*(LM, \mathbb{Q}) \to H_*(L_N M, \mathbb{Q})$ using Hochschild cohomology and study its properties. In particular we extend a result of Félix on the injectivity of the map induced by $\mathrm{aut}_1 M \to \mathrm{map}(N, M; f)$ on rational homotopy groups when M and N have the same dimension and $f : N \to M$ is a map of non zero degree [1].

Keywords: Loop space homology, Poincaré duality, Hochschild cohomology

Category: **Algebra**

References

[1] Y. Félix, *Mapping spaces between manifolds and the evaluation map*, Proc. Amer. Math. Soc. **139** (2011), 3763–3768.

****

Exact Solutions and Conservation Laws of a Alice Bob-KP Equation

Lebogang. Bodibe , **Letlhogonolo .D. Moleleki** & **Masood Khalique**
International Institute for Symmetry Analysis and Mathematical Modelling, Department of Mathematical Sciences, North-West University, Mafikeng Campus, Private Bag X 2046, Mmabatho 2735,
Republic of South Africa
Email: lebo110812@gmail.com, Letlhogonolo.Moleleki@nwu.ac.za, Masood.Khalique@nwu.ac.za

Abstract

In this study a nonlinear evolution partial differential equation, namely, the Alice Bob-KP equation. Lie symmetry method together with $(\frac{G'}{G})$-expansion method is used to find the exact solutions of the Alice Bob-KP equation. Furthermore, multiplier method will be used to construct the conservation laws of the Alice Bob-KP equation.

Keywords: Alice Bob-KP equation, Lie point symmetries, $(\frac{G'}{G})$-expansion method, conservation laws

Category: **other**

References

[1] C. Gu, H. Hu, Z. Zhou, Darboux Transformation in Soliton Theory and Its Geometric Applications, Springer, The Netherlands, 2005.

[2] M.J. Ablowitz, P.A. Clarkson, Solitons, Nonlinear Evolution Equations and Inverse Scattering, Cambridge University Press, Cambridge, UK, 1991.

[3] R. Hirota, The Direct Method in Soliton Theory, Cambridge University Press, Cambridge, 2004.

[4] S.K. Liu, Z.T. Fu, S.D. Liu, Q. Zhao, (2001): Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations, Phys. Lett. A. 289 69-74.

****

On the partial computation of the Lie bracket structure of the string homology on a formal elliptic space

Oteng Maphane[a] & **Jean-Baptiste Gatsinzi**[b]

[a] Botswana International University of Science and Technology
maphaneot@biust.ac.bw

[b] Botswana International University of Science and Technology
gatsinzij@biust.ac.bw

Abstract

We consider the Chas-Sullivan loop space homology $\mathbb{H}_*(X^{S^1})$ of a formal elliptic space X of which $\pi_*(X) \otimes \mathbb{Q}$ is finite dimensional. We show that the centre of the graded Lie algebra $s\mathbb{H}_*(X^{S^1}; \mathbb{Q})$ is non trivial.

Keywords: Sullivan algebra, Loop space homology, formal elliptic spaces

Category: **Algebra**

References

[1] M. Chas, D. Sullivan, *String topology*, Preprint math GT/9911159, (1999).

[2] Y. Félix, S. Halperin, J.C. Thomas, *Rational Homotopy Theory*, Springer, New York, (2001).

****

Rational Homotopy Type of the components of mapping spaces from odd spheres to Complex Grassmannian

Smegnsh Demelash Yeruk[a], **Jean Baptiste Gatsinzi**[b]

[a] Addis Ababa University
smegnshdem@gmail.com

[b] Botswana International Science and Technology University
gatsinzij@biust.ac.bw

Abstract

By considering a certain map $f : X = S^{2(n-k)+1} \times S^{2(n-k)+3} \times \cdots \times S^{2n-1} \to Y = G_{k,n}$, we have shown that the rational homotopy type component of mapping space $map(X, Y; f)$ is product of odd spheres and p Elinberg MacLane space, where $p = 1, \cdots, k-1$ and $G(k, n)$ is the space of k dimensional subspaces of n dimensional complex coordinate space $\mathbb{C}^n$.

Keywords: Function Spaces, rational homotopy type, Sullivan Model, L_∞-algebra,Homogeneous spaces

Category: **Algebra**

References

[1] Buijs, U., Félix, Y. & Murillo, A (2013). *L_∞ rational homotopy of mapping spaces*. Rev Mat Complut 26, 573–588. https://doi.org/10.1007/s13163-012-0105-z

[2] Davis, J. F., Kirk, P. (2001). *Lecture Notes in Algebraic Topology (Vol. 35)*. Providence, Rhode Island : American Mathematical Society. ISBN: 9781470420888 1470420880 978-0-8218-2160-2.

[3] D. Sullivan, *Infinitesimal computations in topology*, Publications Mathématiques de ĹInstitut des Hautes Scientifiques No. 47 (1977), 269–331 (1978). https://doi.org/10.1007/BF02684341

[4] Dieck, Tammo tom(2008), *Algebraic Topology / Tammo Tom Dieck*, Algebraic Topology, Zürich: European Mathematical Society Publishing House, Cop., ISBN: 9783037190487

****

3 Analysis: Special Session

A Tseng Type Algorithm for Approximation of Solution of Equilibrium Problem, Variational Inequality Problem and Fixed Point Problem in Banach Spaces

Bashir Ali & A. M. Hamza

Abstract

In this work, we study a Tseng‘s type algorithm for approximation solution of equilibrium problem, variational inequality problem and fixed point problem in the setting of uniformly smooth which is also 2-uniformly convex real Banach space and 2-uniformly smooth real Banach space and prove its strong convergence to a solution of a variational inequality problem for a monotone $l-$Lipschitz continuous map whose image under a bounded linear operator is a fixed point of nonexpansive maps. The result in this paper is an extension or modification of some recently announce results.

****

Approximation method for monotone inclusion problems in real Banach spaces with applications

Abubakar Adamu[a], **Duangkamon Kitkuan**[b], **Poom Kumam**[a], **Anantachai Padcharoen**[b] & **Thidaporn Seangwattana**[c]

[a] King Mongkut's University of Technology Thonburi, Bangkok 10140, Thailand
aadamu@aust.edu.ng

[b] Rambhai Barni Rajabhat University, Chanthaburi 22000, Thailand

[c] King Mongkut's University of Technology North Bangkok, Rayong, 21120, Thailand

Abstract

In this paper, we introduce an inertial Halpern-type iterative algorithm for approximating a zero of sum of two monotone operators in the setting of real Banach spaces that are 2-uniformly convex and uniformly smooth. Strong convergence of the sequence generated by our proposed algorithm is established by means of some *new* geometric inequalities proved in this paper that are of independent interest. Furthermore, numerical simulations in image restoration and compressed sensing problems are also presented. Finally, the performance of the proposed method is compared with that of some existing methods in the literature.

Keywords: monotone; convex minimization; zeros; image restoration; signal recovery.

Category: **Analysis**

****

Structure results for real Interpolation spaces

Ali Raza
Abdus Salam School of Mathematical Sciences,
GC University Lahore, Pakistan
aliraza@sms.edu.pk

Abstract

The theory of interpolation for quasi-normed spaces was developed in the early 1960s with the work of J. L. Lions, Peetre, A.P. Caldern, and other authors and now it is a large branch of Functional Analysis with many applications to Analysis, Partial Differential Equations, Mathematical Physics.

In this talk, I will give a brief introduction about the Interpolation spaces, there will be two parts of the talk, in first part I will present the basic idea of interpolation theory and then after defining $K-$functional the $K-$method of real interpolation spaces will be discussed in detail. After that, I will discuss some structural results for real interpolation spaces including Holmstedt's formulae and reiteration for different couples formed by interpolation spaces.

Keywords: Interpolation spaces, $K-$ functional, Holmstedt's formulae, Reiteration

Category: **Analysis**

References

[1] Ahmed, I., Edmunds, D. E., Evans, W. D. and Karadzhov, G. E., *Reiteration theorems for the K interpolation method in limiting cases*, Math. Nachr. **284** (2011), 421-442.

[2] Ahmed, I., Karadzhov, G. E. and Raza, A., *Reiteration in $K-$interpolation method for quasinormed spaces*, C. r. de l'Acad. bulgare des Sci. **69** (2016), No 4, 405-410.

[3] Ahmed, I., Karadzhov, G. E., and Raza, A., General Holmstedts Formulae for the-Functional.

[4] Brudnyi, Ju. A. and Krugliak, N. Ja., *Interpolation spaces and interpolation functors*, North- Holland, Amsterdam, 1991.

****

An inertial algorithm for approximating solutions of variational inequality and fixed point problems in reflexive Banach spaces

Bashir Ali[a], **Mohammed Suleman Lawan**[b], **Abdulhadi Muhammead Hamza**[c]

[a] Bayero University, Kano, Nigeria
bashiralik@yahoo.com

[b] Kaduna Polytechnic, Nigeria
mslawankh@yahoo.com

[c] Bayero University, Kano, Nigeria
amhamza.sce@buk.edu.ng

Abstract

In this pepar, we study an inertial extragradient-like S-iteration process for finding a common element of the set of solution of some variational inequality problem involving a monotone Lipschitz map and a fixed point of Bregman demigeneralized mapping in a Banach space. Our result is an extension and generalization of some recently announce results.

Keywords: Strong convergence, Fixed Point Problem, Variational Inequality, Bregman Demi-generalized Mappings.

Category: **Analysis**

****

Existence of fixed points of generalized set-valued $F-$contractions of $b-$metric spaces

Basit Ali[a] & **Aqsa Butt**[a]

[a]Department of Mathematics, School of Sciences, University of Management and Technology, C-II, Johar Town, Lahore, 54770, Pakistan.

E-mail addresses: basit.aa@gmail.com, basit.ali@umt.edu.pk, AqsaButt013@gmail.com

s

Abstract

This paper deals with the existence of nonempty fixed point sets of newly introduced generalized set-valued $F-$contractions of $b-$metric spaces. Some illustrative examples show that the new results in this work generalize properly, unify and extend some related results in the existing literature. Moreover, we extract some important consequences of the results in $b-$metric spaces. Particularly by setting $b-$metric constant equal to one, we obtain some specific cases showing notable enhancement of existing results yet in metric spaces.

Keywords: $b-$metric, fixed points, set-valued mapping, $F-$contractions, almost $F-$contractions.

Category: **Analysis**

References

[1] S. Czerwik, Contraction mappings in b-metric spaces, Acta Math. Inform. Univ. Ostrav., 1 (1993) 5-11.

[2] S. Banach, Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales, Fund. math., 3(1) (1922) 133-181.

[3] D. Wardowski, Fixed points of a new type of contractive mappings in complete metric spaces, Ward. Fixed Point Theory Appl., 94 (2012).

****

New modified inertial projection method for bilevel variational inequality and fixed point problems

Godwin Chidi Ugwunnadi

Department of Mathematics, University of Eswatini, Private Bag 4, Kwaluseni, Eswatini
ugwunnadi4u@yahoo.com

Abstract

In this paper, we introduce a new method for solving bilevel quasimonotone variational inequality and fixed point problems in a real Hilbert spaces. Then, we establish the strong convergence of the proposed method under some mild conditions on the parameters. Finally, we give some numerical experiments to show efficiency of the method over some recently known related methods in the literature.

Keywords: Fixed Point, monotone operator, Variational inequality Problem, Demimetric mapping, Hilbert space

Category: **Analysis**

References

[1] Banos, D.R., Di Nunno, G., Haferkorn, H. H. and Proske, F. (2018): Stochastic Functional Differential Equations and Sensitivity to Their Initial Path. *E. Celledoni, G. Di Nunno, K. Ebrahimi-Fard and H. Munthe-Kaas (eds) Computation and Combinatorics in Dynamics, Stochastics and Control. Abelsymposium 2016. Abel Symposia, vol 13. Springer, Cham*

[2] Benth, F. E. and Simonsen, I. C. (2018):The Heston stochastic volatility model in Hilbert space. *Stochastic Analysis and Applications*.

****

Inertial algorithm for solution of Hammerstein inclusions in Banach spaces

Ibrahim Mohammed[a] & **Ma'aruf Minjibir**[b]

[a]Department of Mathematical Sciences, Abubakar Tafawa Balewa University Bauchi, Nigeria.
mibrahim8@atbu.edu.ng

[b] Department of Mathematical Sciences, Bayero University, Kano, Nigeria.
msminjibir.mth@buk.edu.ng

Abstract

For $q > 1$, let X be a real q-uniformly smooth Banach space. Let $F, K : X \to 2^X$ be strongly accretive, continuous and bounded maps. An inertial algorithm for solutions of inclusions of Hammerstein type, $0 \in u + KFu$, is constructed. Using a path strongly converging to a zero of m-accretive map, the sequence obtained is shown to strongly converge to a solution of the mentioned inclusion, given existence of solution. Our results constitute an improvement of a related result in the setting of Hilbert spaces[Bello, A. U., Omojola, M. T. and Yahaya, J., *An inertial-type algorithm for approximation of solutions of Hammerstein integral inclusions in Hilbert spaces*, Fixed Point Theory Appl. 2021:8. DOI: 10.1186/s13663-021-00691-7] and the corresponding non-inertial algorithm [Minjibir, M.S. and Mohammed, *Solutions for Hemmerstein inclusions*, Pan American Math. J. **30** (2020), no. 2, 37-54].

Keywords: Accretive map, q-uniformly smooth, m- accretive, bounded, Hemmerstein inclusions,

Category: **Analysis**

References

[1] Minjibir, M.S. and Mohammed, I., *Solutions for Hemmerstein inclusions*, Pan American Maths. J. **30** (2020), no. 2, 37-54.

[2] Bello, A. U., Omojola, M. T. and Yahaya, J.,*An inertial-type algorithm for approximation of solutions of Hammerstein integral inclusions in Hilbert spaces,* Fixed Point Theory Appl., 2021:8. DOI: 10.1186/s13663-021-00691-7.

****

Remembering the Astute Non-linear Functional Analysis, Professor Charles Ejike Chidume: 1947-2021.

Jeremiah N. Ezeora[a], [a]Department of Mathematics and Statistics, University of Port Harcourt, Nigeria
jeremiah.ezeora@uniport.edu.ng

Abstract

The Analysis section of the SAMSA/MASAMU meeting and conference, 2022 plans to honor late Professor Charles Ejike Chidume , an astute Non-linear Analyst . This presentation highlights the numerous contributions of the great Mathematician, Professor Chidume to development of Non-linear Functional Analysis . We shall make frantic efforts to bring to fore, his outstanding contributions to Non-linear functional Analysis, the Mathematics Community and humanity in general.

Keywords: Non-linear Analysis, Mathematics Community , SAMSA/MASAMU conference.

Category: **Industiral Mathematics**

****

Fourier transform decay of distributions in Hardy-Morrey spaces

Marcelo F. de Almeida [a]
[a] Federal University of Sergipe, Departamento de Matemática, Brazil
marcelo@mat.ufs.br

Abstract

In this note we establish decay estimates for Fourier transform on Hardy-Morrey spaces, namely,

$$|\widehat{f}(\xi)| \lesssim |\xi|^{n\left(\frac{1}{\lambda}-1\right)} \|f\|_{\mathcal{HM}^{\lambda}_{q}} \quad 0 < p \leq \lambda \leq 1$$

and its localizable version. Our work include some aspects to these spaces linked up with previous estimates, in particular a natural approach on cancellation moment conditions. As application, we discuss the optimality for continuity of Fourier multipliers and pseudodifferential operators in Hardy-Morrey spaces.

Keywords: Atomic Decomposition; Fourier transform decay; Hardy-Morrey spaces.

Category: **Analysis**

References

[1] M. Bownik and L.-A. D. Wang, Fourier transform of anisotropic Hardy spaces, Proc. Amer. Math. Soc. **141** (2013), no. 7, 2299–2308.

[2] J. Hounie and R. Kapp, *Pseudodifferential Operators on Local Hardy Spaces*, J. Fourier Anal. Appl. **15** (2009), 153–178.

[3] H. Jia and H. Wang, *Decomposition of Hardy-Morrey spaces*, J. Math. Anal. Appl. **354** (2009), no. 1, 99–110.

****

Evaluating and estimating some important Integrals using distribution or decreasing rearrangement functions and the Lorentz space $L(p,1), p>1$

Nozara Sundus[a], **Eddy Kwessi**[b] & **Geraldo de Souza**[c]
[a] Abdus Salam School of Mathematical Sciences, Government College University, Lahore 54600, Pakistan
nozarasundus21@sms.edu.pk

[b] Trinity University, One Trinity Place, San Antonio, TX 78212, USA
ekwessi@trinity.edu

[c] Department of Mathematics and Statistics, Auburn University, 221 Parker Hall, Auburn, AL 36849, USA
desougs@auburn.edu

Abstract

In this talk we explore the famous relationship between Lp-norm in terms of distribution and decreasing rearrangement functions to evaluate some important higher dimensions integral, indeed we use

$$\int \left|f\left(x\right)\right|^{p} d\mu\left(x\right) = p\int_{0}^{\infty} \alpha^{p-1}\mu_{f}\left(\alpha\right) d\alpha = \int_{0}^{\infty} \left(f^{*}\left(t\right)\right)^{p} dt.$$

The first integral on left is on X, the measures space $(X, \sigma$-algebra on $X, \mu)$ for $p > 0, \mu_f$ the distribution function and f^* the decreasing rearrangement of f.

Finally we look the Lorentz spaces $L(p, 1)$ for $p \geq 1$ in 2-dimension.

Which says $f \in L(p, 1)$ if and only if $\|f\|_{(L(p,1))} = \int_0^\infty f^*(t)\, t^{\frac{1}{p}-1} dt < \infty$, and we give a new characterization based on special atom spaces, and this allow us to study some operators on the Lorentz spaces $L(p, q), p, q > 0$. We will comment on important facts about $L(p, q)$.

Keywords: Integrals, Distribution function, Decreasing rearrangement function, Lorentz Space

Category: **Analysis**

References

[1] Kwessi, E., De Souza, G., Djitte, N. and Ndiaye, M. (2020): The special atom space and Haar wavelets in higher dimensions. Demonstratio Mathematica, *53(1), 131-151.*

[2] Grafakos, L. (2008): Classical fourier analysis. vol. 2. Springer, New York.

****

Fixed point approximation schemes in geodesic spaces

Sani Salisu[a], **Ma'aruf Shehu Minjibir**[b], **Poom Kumam**[c] & **Songpon Sriwongsa**[d]

[a] King Mongkut's University of Technology Thonburi, Thailand
sani.salisu@mail.kmutt.ac.th

[b] Bayero University Kano, Nigeria
msminjibir.mth@buk.edu.ng

[c] King Mongkut's University of Technology Thonburi, Thailand
poom.kum@kmutt.ac.th

[d] King Mongkut's University of Technology Thonburi, Thailand
songpon.sri@kmutt.ac.th

Abstract

In this talk we present geodesic convexity as a tool for fixed point theory and convex analysis in metric spaces. We discuss a generalised class of nonexpansive mappings in a metric space of special geometry. In addition, we explore JK-iteration as a scheme of approximating fixed point of such mappings and achieve fixed point convergence results using demiclosedness-type property.

Keywords: $\text{CAT}_p(0)$ spaces, demiclosedness-type property, fixed point, geodesic spaces, JK-iteration, nonexpansive mapping.

Category: **Analysis**

References

[1] Salisu, S., Minjibir, M.S., Kumam, P., and Sriwongsa, S. (2022): Convergence theorems for fixed points in $\text{CAT}_p(0)$ spaces. *Journal of Applied Mathematics and Computing, 1-20*.

****

An overview of the m-splitting theory

Yaé Ulrich Gaba[a],
[a] University of Limpopo, South Africa
yae.gaba@ul.ac.za

Abstract

We present the results of our investigations on those (partially) ordered metric spaces $(X, m, \leq)$ for wich there exists an m-splitting T_0-quasi-metric d on X such that $x \leq y$ if and only if $d(x, y) = 0$ for any $x, y \in X$. In particular, for a given metric space (X, m) we present results about T_0-quasi-metrics on X that are minimal among the collection of all m-splitting T_0-quasi-metrics on X. We give the current state of the art, describe work-in-progress and suggest questions we think could be interesting for further investigations.

Keywords: Metric space, Partial order, Quasi-pseudometric, Group, Lattice, Splitting a metric, Minimally splitting.

Category: **Topology; Algebra & Analysis**

References

[1] Conradie, J., & Künzi, H. P. A. (2018). Asymmetric norms given by symmetrisation and specialisation order. *Topology and its Applications, 242, 1-19*.

[2] Gaba, Y. U., & Künzi, H. P. A. (2015). Splitting metrics by T_0-quasi-metrics. *Topology and its Applications, 193, 84-96*.

[3] Gaba, Y. U., & Künzi, H. P. A. (2016). Partially ordered metric spaces produced by T_0-quasi-metrics. *Topology and its Applications, 202, 366-383*.

****

Inertial Version of Generalized Projected Reflected Gradient Method

Yekini Shehu[a]

[a] College of Mathematics and Computer Science, Zhejiang Normal University, Jinhua 321004, People's Republic of China

yekini.shehu@zjnu.edu.cn

Abstract

In this talk, we study a generalized version of projected reflected gradient method coupled with inertial extrapolation step to solve variational inequalities in Hilbert spaces. Our proposed method requires one function evaluation and one projection per iteration alongside inertial extrapolation step which is motivated by the desire to devise faster and less computationally expensive iterative methods for variational inequalities. We obtain weak and linear convergence of the sequence of iterates generated by our method under some standard conditions and numerical results are given to show the efficacy of the proposed iterative scheme. Several versions of recently proposed projected reflected gradient methods in the literature are recovered from our method.

Keywords: variational inequalities; projected reflected gradient method; inertial steps; weak and linear convergence; Hilbert spaces.

2020 MSC classification: 47H05, 47J20, 47J25, 65K15, 90C25.

****

4 Analysis

Solution of System of Klein-Gordon Equations using Reduced Differential Transform Method

Albert Bii[a] & **Julius Maremwa Shichikha**[b]

[a]University of Eldoret, Department of Mathematics and Computer Science, P. O. Box 1125-30100, Eldoret-Kenya

albert.bii@uoeld.ac.ke

[b] University of Eldoret, Department of Mathematics and Computer Science, P. O. Box 1125-30100,Eldoret-Kenya

jmaremwa@yahoo.com/julius.maremwa@uoeld.ac.ke

Abstract

Real-world models are formulated using partial differential equations. Klein-Gordon Equations (KGE) [1] describe the interactions of sub-atomic particles. Numerical solutions are available but not accurate [2]. In this paper, we develop a fourth order scheme using Reduced Differential Transform Method [3]. The method transforms the KGE into a differential recurrence relation from which convergent series can be constructed to give a better solution. The solutions are then presented graphically.

Keywords: Reduced Differential Transform Method, Convergent series, Klein-Gordon.

Category: **Mathematical Physics**

References

[1] Ita, B. I. and Tchoua, P. (2013).The Solution of Klein-Gordon Equation using Reduced Differential Transform Method. *Research Journal of Physical Sciences*. Volume 1(1) pp 21–23.

[2] Keskin, Y. S. and Oturanc, G. (2011). Reduced Differential Transform Method for Solving Klein-Gordon Equations. *Proceedings of the World Congress on Engineering*. Volume 1, pp 6–8. London, U. K: World Congress on Engineering.

[3] Mohammad, H. et al. (2017) *Differential Transformation Method for Mechanical Engineering Problems*. Academic Press.

****

The Jacobson Property in Banach algebras

André Swartz[a] & **Hein Raubenheimer**[b]

[a] University of Johannesburg

aswartz@uj.ac.za

[b] University of Johannesburg

heinrichr@uj.ac.za

Abstract

In this talk A will represent a non commutative and unital Banach algebra. We let $\sigma(a)$ represent the usual spectrum of $a \in A$. It is well known that for $a, b \in A$ we have

$$\lambda \in \sigma(ab) \setminus \{0\} \iff \lambda \in \sigma(ba) \setminus \{0\}.$$

We are interested in subsets of A that have the *Jacobson Property*, i.e. $X \subset A$ such that for $a, b \in A$:

$$1 - ab \in X \iff 1 - ba \in X.$$

Keywords: Functional Analysis, Spectral Theory, Regularity, Semiregularity

Category: **Analysis**

****

Lagrangian formulation of a Lane-Emden-Klein-Gordon-Fock system with central symmetry

Ben Muatjetjeja[a,b]

[a] Department of Mathematics, Faculty of Science, University of Botswana, Private Bag 22, Gaborone, Botswana
muatjetjejab@ub.ac.bw

[b] Department of Mathematical Sciences, North-West of University, Private Bag X2046, Mmabatho 2735, South Africa

Abstract

In this talk we introduce a sort of Lane-Emden system derived from the Klein-Gordon- Fock equation with central symmetry. Point symmetries are obtained and, since the system can be derived as the Euler-Lagrange equation of a certain functional, a Noether symmetry classification is also considered and conservation laws are derived from the point of Noether symmetries.

Keywords: Lane-Emden-Klein-Gordon-Fock; Noether symmetries; Conservation laws.

References

[1] G.M. Moatimid, R.M. EI-Shiekh, A.A.A.H. AI-Nowehy, Exact solutions for Calogero-Bogoyavlenskii-Schiff equation using symmetry method, Appl. Math. Comput. 220 (2013) 455-462

[2] Muatjetjeja, B. Coupled Lane-Emden-Klein-Gordon-Fock system with central symmetry: Symmetries and Conservation laws. *J. Differ. Equ. 263 (2017) 8322–8328.*

[3] Freire, I.L. and Muatjetjeja, B. Symmetry analysis of a Lane-Emden-Klein-Gordon-Fock system with central symmetry. *Discrete. Contin. Dyn Syst. Ser A. 11 (2018) 667–673.*

****

Projection and contraction method for solving quasi-monotone variational inequality with two-step inertial

Chibueze C. Okeke
School of Mathematics, University of the Witwatersrand,
Private Bag 3, Johannesburg 2050, South Africa
chinedu.izuchukwu@wits.ac.za

Abstract

In this article, Projection and contraction method with two- step inertial extrapolation and self-adaptive step sizes is proposed to solve variational inequalities in quasi-monotone setting. A two-step inertial extrapolation is added to further improve on the convergence speed of the proposed method and self-adaptive step sizes are used in order to reduce computational complexity of our method. Weak convergence analysis are obtained under some easy to verify conditions on the iterative parameters in Hilbert spaces.

****

Boundedness Results for Solutions of Certain Second Order Non-Autonomous Ordinary Differential Equations

Daniel Oluwasegun Adams & **Mathew Omonigho Omeike**
Department of Mathematics, Federal University of Agriculture, Abeokuta, Nigeria
adamsdo@funaab.edu.ng

Abstract

We shall consider the second order non-autonomous nonlinear ordinary differential equations:

$$a(t)x'' + b(t)f(x;x') + c(t)[g(x') + m(x)]h(x') = p(t;x;x')$$

and

$$(a(t)x')' + b(t)f(x;x') + c(t)[g(x') + m(x)]h(x') = p(t;x;x'),$$

where a, b, c, f, g, m, h and p are real valued functions which depend at the most on the argument displayed explicitly. In this paper, we shall use different forms of integral inequalities and two forms of mean value theorem for integrals to investigate the boundedness of all solutions and their derivatives.

Keywords: boundedness; nonlinear; second order; integral in-equalities; differential equations

****

On periodic self-adjoint spectral problem for a functional-differential equation of even order

Elena Alves[a], **Manuel Alves**[b] & **Sergey Labovskiy**[c]
[a] Universidade Politecnica, Escola Superior de Gestao e Tecnologias, Av. Paulo Samuel Kamkonba 1170, Maputo, Mozambique
elena.vlad.alves@gmail.com

[b] Eduardo Mondlane University, Faculty of Sciences, Main Campus, Maputo, Mozambique
mjalves.moz@gmail.com

[c] Plekhanov Russian University of Economics
labovski@gmail.com

Abstract

The study of the spectra of differential operators is connected with applications in mechanics and quantum mechanics. First of all, the second-order operator has been well studied due to its self-adjointness due to symmetry in physics. A generalization is an operator of even order, in which self-adjointness is preserved. In [1], a self-adjoint functional-differential operator ($:=$ means *'is equal to' by definition*)

$$\mathcal{L}u(x) := \frac{1}{\rho(x)}\left((-1)^m u^{(2m)} - \int_0^l u(s)q(x,ds)\right), \quad x \in [0,l] \quad (m \geq 1) \tag{1}$$

($\rho(x)$ is a fixed positive weight function) is studied, the symmetry of which is determined by the properties of the function $q(x,ds)$. In paper [2], is studying the periodic boundary value problem

$$-(p(x)u')' + R(x)u - \int_0^l u(s)d_s r(x,s) = \lambda\rho(x)u \tag{2}$$

$$u(0) = u(l),\ pu'|_{x=0} = pu'|_{x=l}. \tag{3}$$

Function $r(x,\cdot)$ is assumed to be non-decreasing, and $r(x,0) = 0$. In the second condition, expression pu' is considered as a quasi-derivative of function u.

In the presented work, we study the operator

$$\mathcal{L}u(x) := \frac{1}{\rho(x)}\left((-1)^m u^{(2m)} + R(x)u - \int_0^l u(s)q(x,ds)\right), \quad x \in [0,l] \quad (m \geq 1) \tag{4}$$

but with periodic boundary conditions

$$u^{(k)}(0) = u^{(k)}(l),\ k = 0,\dots,m-1, \tag{5}$$

and

$$u^{(k)}(0) = u^{(k)}(l),\ k = m,\dots,2m-1, \tag{6}$$

and a condition for $q(x,ds)$ the operator $\mathcal{L}$ will be self-adjoint. These periodic conditions are divided into two groups due to the fact that the first group determines the space for research using the variational method. The periodic boundary value problem is important for applications in mechanics and mathematical physics, its mechanical interpretation suggests the properties of solutions to the boundary value problem.

Let $\Delta := [0,l]$, $L_2(\Delta,\rho)$ be the space of Lebesgue quadratic integrable on Δ with positive weight $\rho(x)$ and scalar product

$$(f,g) := \int_0^l f(x)g(x)\rho(x)\,dx,$$

$L_2(\Delta) := L_2(\Delta,1)$. Assume that $\int_0^l \rho(x)\,dx < \infty$. Let AC^k ($k \geq 0$) be the set of functions u that have absolutely continuous on $[0,l]$ derivative $u^{(k)}$, $u^{(0)} := u$, with the norm

$$\|u\|_{AC^k} = \max_{0\leq i\leq k}\left|u^{(i)}(0)\right| + \int_0^l \left|u^{(k+1)}(x)\right| dx.$$

Introduce the bilinear form

$$[u,v] := \int_0^l \left(u^{(m)}(x)v^{(m)}(x) + R(x)uv \right) dx - \int_0^l dx\, v(x) \int_0^l u(s) d_s r(x,s).$$

Let W is the Hilbert space of functions in AC^{m-1}, satisfying the conditions (5) and $[u,u] < \infty$, with scalar product $[u,v]$. Let $T\colon W \to L_2(\Delta,\rho)$ be the operator defined by $Tu(x) := u(x)$, $x \in \Delta$.

Theorem 1. *The spectral problem $\mathcal{L}u = \lambda Tu$ under boundary conditions (5),(6) has complete and orthogonal in $L_2(\Delta,\rho)$ system of eigenfunctions: $\mathcal{L}u_k = \lambda_k Tu_k$, $k = 0,1,2,\ldots$. The eigenvalues are bounded from below and have the unique density point $+\infty$, that is, $\lambda_0 \le \lambda_1 \le \cdots$, and $\lambda_k \to \infty$. If λ is not an eigenvalue, the BVP $\mathcal{L}u - \lambda Tu = f$ has unique solution in W for any $f \in L_2(\Delta,\rho)$.*

Keywords: quadratic functional, spectrum, positive solution

Category: **Analysis**

References

[1] M. Alves and S. Labovskiy, *On monotone solutions and a self-adjoint spectral problem for a functional-differential equation of even order*, Electronic Journal of Qualitative Theory of Differential Equations, 2019 (2019), pp. 1–14.

[2] M.Alves and S.Labovskiy, *On the spectral properties and positivity of solutions of a second-order functional differential equation*, Russian Universities Reports. Mathematics, 25 (2020), pp. 123–130. https://elibrary.ru/item.asp?id=43145436, doi =10.20310/2686-9667-2020-25-130-123-130.

****

Inertial Extrapolation Method for Solving Split Common Fixed Point Problem and Zeros of Monotone Operators in Hilbert Spaces

L. Y. Haruna[a], **G. C. Ugwunnadi**[b], **B. Ali**[c],
[a] Department of Mathematical Sciences, Kaduna State University, Kaduna-Nigeria
yulah121@gmail.com

[b] Department of Mathematics, University of Eswatini, Private Bag 4, Kwaluseni, Eswatini
ugwunnadi4u@yahoo.com

[c] Department of Mathematical Sciences, Bayero University, Kano, Kano-Nigeria
bashiralik@yahoo.com

Abstract

In this paper, we introduce a hybrid-type proximal point algorithm for approximating zero of monotone operator in Hadamard-type spaces. We then prove that a sequence generated by the algorithm involving Mann-type iteration converges strongly to a zero of the said operator in the setting of flat Hadamard spaces. To the best of our knowledge, this result presents the first hybrid-type proximal point algorithm in the space. The result is applied to convex minimization and fixed point problems.

Keywords: Fixed Points, Proximal point algorithm, Hybrid-Type algorithm, Flat Hadamard spaces

Category: **Analysis**

References

[1] Banos, D.R., Di Nunno, G., Haferkorn, H. H. and Proske, F. (2018): Stochastic Functional Differential Equations and Sensitivity to Their Initial Path. *E. Celledoni, G. Di Nunno, K. Ebrahimi-Fard and H. Munthe-Kaas (eds) Computation and Combinatorics in Dynamics, Stochastics and Control. Abelsymposium 2016. Abel Symposia, vol 13. Springer, Cham*

[2] Benth, F. E. and Simonsen, I. C. (2018):The Heston stochastic volatility model in Hilbert space. *Stochastic Analysis and Applications*.

****

Properties of Bishop - Phelps cone in a Banach space

Mokhwetha Mabula[a] & **Sello Mbambo**[b]

[a] University of Pretoria, South Africa
mokhwetha.mabula@up.ac.za

[b] University of the Free State, South Africa
mbambosp@ufs.ac.za

Abstract

In this talk, we study some properties of a Bishop - Phelps cone (BP - cone for short). This cone is of this form:

$$C_f = \{x \in X : \|x\| \leq f(x)\} \text{ where } X \text{ is a normed space and } f \in X^*, \text{ dual of } X.$$

The existence of a non - solid BP cone, C_f, in a non - reflexive Banach space is investigated. We present the partial result of I. Polyrakis et. al to the question posed by Qui for separable non - reflexive spaces with different proof using BP cones. Finally the extention to non - separable spaces is been considered.

Category: **Analysis**

References

[1] I.A. Polyrakis, F. Xanthos, *Cone characterization of Grothendieck and Banach spaces containing* c_0, Positivity, 2011, 15: 677 - 693.

[2] J.H. Qui, *A cone characterization of reflexive Banach space,*Journal of Mathematics Analysis and Applications, 2001, 256: 39 - 44.

****

Highly accurate multi-domain multivariate spectral collocation method for (2+1) dimensional Burger's equations

Musawenkhosi P. Mkhatshwa
Department of Mathematical Sciences, University of South Africa Cnr Christian de Wet Rd
and
Pioneer Avenue, Florida Park, 1709, Roodepoort, South Africa.
patsonmkhatshwa@gmail.com

Abstract

The novelty of this work rests upon the use of the domain partitioning technique in time variable when discretizing the domain of solution in spectral collocation algorithm. The single domain multivariate spectral collocation-based methods have been proven to be effective in solving time-dependent partial differential equations (PDEs) defined over small time domains. However, there is a significant loss of accuracy as time computational domain proliferates and also when the number of grid nodes approaches a definite particular number. Therefore, the establishment of the new innovative multi-domain multivariate spectral quasilinearisation method (MDMV-SQLM) is described for the purpose of solving (2+1) dimensional nonlinear PDEs defined on large time intervals. The main output of this study is confirmation that minimizing the size of time computational domain at each subinterval assures sufficiently accurate results that are attained using minimal number of nodal points and less computational time. To highlight the efficiency and accuracy of the MDMV-SQLM, error estimates, condition numbers and computational time are presented for well known (2+1) dimensional nonlinear Burger's PDEs. The adoption of the domain decomposition technique is efficacious in suppressing the numerical challenges linked to large matrices and ill-conditioned nature of the resulting coefficient matrix. Also, the obtained results confirm that the numerical scheme is computationally cheap, fast and yield extremely accurate and stable results using fewer number of grid points for large time domains.

Keywords: Multi-domain, Multivariate spectral quasilinearisation method, (2+1) dimensional Burger's equations, Trivariate Lagrange interpolation, Kronecker tensor product

Category: **Analysis**

****

Developing Higher order Unconditionally Positive Finite Difference Method for the Advection Diffusion Reaction Equations

Ndivhuwo Ndoua, **Phumlani Dlamini** & **Byron Alexander Jacobs**
Department of Mathematics and Applied Mathematics, University of Johannesburg, P.O. Box 524, Auckland Park 2006, South Africa;
220164883@student.uj.ac.za (N.N), pgdlamini@uj.ac.za (P.D), byronj@uj.ac.za (B.A.J)

Abstract

In this paper, higher order unconditionally positive finite difference (HUPFD) methods are developed to solve linear and non-linear advection-diffusion-reaction (ADR) equations. The stability and consistency of the developed methods are analyzed, which are necessary and sufficient for convergence to the exact solution. The Von Neuman condition is used to analyze the stability since we are dealing with the Cauchy problem. The proposed method's efficiency and effectiveness is investigated by calculating the error, convergence rate, and computational time. A comparison of the solutions obtained by the higher order unconditionally positive finite difference and analytical methods is conducted for validation purposes. The numerical results show that the developed method preserve the solution accuracy. The results also show that increasing the order of the unconditionally positive finite difference leads to the implicit scheme that is unconditionally stable with an increased order of accuracy with respect to time and space..

Keywords: Higher order unconditionally positive finite difference method, unconditionally positive finite difference method, advection diffusion reaction equations, Von Neuman stability analysis, Consistency and Stability analysis.

Category: **Analysis**

References

[1] Calvo, M., De Frutos, J. & Novo, J. Linearly implicit RungeKutta methods for advection-reactiondiffusion equations. *Applied Numerical Mathematics*. **37**, 535-549 (2001)

[2] Khan, H., Gmez-Aguilar, J., Khan, A. & Khan, T. Stability analysis for fractional order advectionreaction diffusion system. Physica A: Statistical Mechanics And Its Applications. **2019.** 521 pp. 737-751.

[3] Zhao, G. & Ruan, S. Time periodic traveling wave solutions for periodic advectionreactiondiffusion systems. Journal Of Differential Equations. **2024** 257, 1078-1147.

[4] Dwivedi, K., Das, S. & Baleanu, D. Numerical solution of nonlinear spacetime fractionalorder advectionreactiondiffusion equation. *Journal Of Computational And Nonlinear Dynamics.* **2020** 15, 061005.

****

An integral deferred correction method for two-point boundary value problems

S'yanda Mungwe
University of Stellenbosch
mungwes@sun.ac.za

Abstract

Ordinary and partial differential equations, and in particular boundary value problems (BVPs) and initial-boundary value problems, are ubiquitous in applications from science and engineering. Numerical approximations are typically required, usually with some kind of method of lines approach to separate the time and space variable discretisations. If high accuracy is required, then spectral methods can be used to discretise the spatial part. There are many techniques for finding solution of BVPs, such as finite difference methods, shooting methods and finite elements, but these are typically low accuracy in space. Greengard et al. introduced in 2000 the idea of spectral deferred correction (SDC), which allows an efficient way to get spectral accuracy (geometric convergence) in time. The method achieves the spectral accuracy in time through the correction function developed using Picard integral equation, repeatedly correcting errors from a low order method to obtain high order accuracy. In this talk, we discuss the integral deferred correction (IDC) approach for BVPs. Rather than using the Picard integral equation, IDC uses a function developed from implicit defect errors to repeated correct errors from finite difference methods. It is through using quadrature rule in error function that the methods achieve high order accuracy. IDC is proposed as efficient way to get the spectral accuracy in space due to the sparsity of matrix applied in 'backslash' in each correction.

Keywords: Integral deferred correct, Boundary value problems

Category: **Scientific Computing and Numerical Mathematics**

References

[1] A. Dutt, L. Greengard and V. Rokhlin, *Spectral deferred correction method for ordinary differential equations*, BIT, 40 (2000), pp. 241-266.

****

Symmetry classification of a coupled system of ordinary differential equations

Sivenathi Mbusi[a], **Ben Muatjetjeja**[a,b] & **Abdullahi Rashid Adem**[c]
[a] North West University, Mafikeng, South Africa
sivenathi.mbusi@nwu.ac.za

[b] Department of Mathematics, Faculty of Science, University of Botswana, Private Bag 22, Gaborone, Botswana

muatjetjejab@ub.ac.bw

[c] Department of Mathematical Sciences, University of South Africa, UNISA, Pretoria 0003, South Africa
ademar@unisa.ac.za

Abstract

We perform a complete symmetry analysis of a generalized Lane-Emden-Klein-Fock system with central symmetry. Several cases for the non-equivalent forms of the arbitrary elements are obtained. Moreover, a symmetry reduction for some cases is performed.
Keywords: Lie group classification; Equivalent transformation; Lie point symmetries; similarity reduction.

Category: **Analysis**

References

[1] S.O. Mbusi, B. Muatjetjeja A.R. Adem, Lie Group Classification of a Generalized Coupled Lane-Emden-Klein-Gordon-Fock system with central symmetry, *Nonlinear Dyn. Syst. Theory. 19 (2019) 186–192.*

[2] Muatjetjeja, B. Coupled Lane-Emden-Klein-Gordon-Fock system with central symmetry: Symmetries and Conservation laws. *J. Differ. Equ. 263 (2017) 8322–8328.*

[3] Freire, I.L. and Muatjetjeja, B. Symmetry analysis of a Lane-Emden-Klein-Gordon-Fock system with central symmetry. *Discrete. Contin. Dyn Syst. Ser A. 11 (2018) 667–673.*

****

Sets with the Baire property in topologies defined from Vitali selectors of the real line

Venuste NYAGAHAKWA
University of Rwanda
venustino2005@yahoo.fr

Abstract

Let V be a Vitali selector related to a countable dense subgroup Q of the additive topological group $(\mathbb{R}, +)$ of real numbers. In this paper, we discuss different properties of the space $(\mathbb{R}, \tau(V))$, where $\tau(V)$ is a topology having the collection $\{V + q : q \in Q\}$ as a base. We prove that if W is a Vitali selector related to a countable dense subgroup P of $(\mathbb{R}, +)$ different from Q, then the spaces $(\mathbb{R}, \tau(V))$ and $(\mathbb{R}, \tau(W))$ are homeomorphic. As a consequence, the family of sets having the Baire property in $(\mathbb{R}, \tau(V))$ and the family of sets with

the Baire property in $(\mathbb{R}, \tau(W))$ are Baire congruent; that is, there exist bijective mappings $f : (\mathbb{R}, \tau(V)) \longrightarrow (\mathbb{R}, \tau(W))$ and $g : (\mathbb{R}, \tau(W)) \longrightarrow (\mathbb{R}, \tau(V))$ which preserve the Baire property. We further look at the Baire congruence relation between the families of sets with the Baire property in $*$-topologies defined from $\tau(V)$ by using ideals of sets in Janković and Hamlett sense.

Keywords: Baire Property, Baire congruent, Vitali selectors, Partition Topology.

Category: **Analysis**

References

[1] Kharazishvili A. B., *Nonmeasurable sets and functions*, Elsevier, Vol.195, 2004.

[2] Oxtoby J. C., *Measure and category: Graduate texts in mathematics*, Springer-Verlag, New York, Heldenberg, Berlin, 1971.

[3] Kharazishvili A. B., *Measurability properties of Vitali sets*, Amer. Math. Monthly. 118 (2011), No. 8, 693 - 703.

[4] Janković D. J. and Hamlett T. R., New topologies from old via Ideals. Amer. Math. Monthly. 97 (1990), No. 4, 295-310.

****

Common Attractors of Generalized Iterated Function Systems

Vuledzani Makhoshi[a], **Talat Nazir**[b], **Melusi Khumalo**[c]
University of South Africa
[a] vuledzani.makhoshi@univen.ac.za
, [ab] talatn@unisa.ac.za
, [c] khumam@unisa.ac.za

Abstract

In this talk, we develop some some new common attractors with the assistance of finite families of generalized contractive mappings, that belong to the special class of mappings defined on a partial metric space. Consequently, a variety of results for iterated function systems satisfying a different set of generalized contractive conditions are acquired. We present some examples to reinforce the results proved herein. These results generalize, unify and extend a variety of results that exist in current literature.

****

An inertial method for Split Equality Common f, g-fixed point problems of f, g-pseudocontractive mappings in Reflexive real Banach Spaces

Yirga Abebe Belay[a], **Habtu Zegeye Hailu**[b] & **Oganeditse A. Boikanyo**[c]
[a] Botswana International University of Science and Technology
yirga2006@gmail.com
[b] Botswana International University of Science and Technology
habtuzh@yahoo.com
[c] Botswana International University of Science and Technology
boikanyoa@gmail.com

Abstract

Let E_1, E_2, and E_3 be reflexive real Banach spaces with duals E_1^*, E_2^*, and E_3^*, respectively. The split equality common $f, g-$fixed point problem (SECFPP) is defined as finding a point $(x^*, y^*) \in E_1 \times E_2$ such that

$$(x^*, y^*) \in (F_f(T_1) \cap F_f(T_2)) \times (F_g(S_1) \cap F_g(S_2)) : Ax^* = By^*, \tag{7}$$

where f: $E_1 \to (-\infty, +\infty]$ and g: $E_2 \to (-\infty, +\infty]$ are proper, lower-semi-continuous and Gâteaux differentiable convex functions, T_1, T_2: $E_1 \to E_1^*$ are $f-$pseudocontractive and S_1, S_2: $E_2 \to E_2^*$ are $g-$pseudocontractive mappings, A: $E_1 \to E_3$ and B: $E_2 \to E_3$ are bounded linear mappings. The notation $F_f(T)$ stands for the set of $f-$fixed points of a mapping T.

This problem is connected with many problems in nonlinear analysis and optimization such as convex minimization problems, split feasibility problems, variational inequality problems and equilibrium problems. Due to its diverse applications in economics, medicine, and engineering, the techniques and methods for solving (7) have received much attention.

This study introduced an inertial method for solving Split Equality Common $f, g-$ Fixed Point Problem (SECFPP), which involves $f, g-$pseudocontractive mappings in reflexive real Banach spaces. A strong convergence theorem is established under the assumption that the mappings under consideration are uniformly continuous $f, g-$pseudocontractive. A numerical example is also provided to demonstrate the effectiveness of the proposed algorithm.

The results in this study generalize and improve many of the results in the literature in the sense that the underlying spaces are reflexive Banach spaces which are more general than Hilbert spaces.

Keywords: Banach spaces, Bregman distance, $f-$fixed points, $f-$pseudocontractive mapping, maximal monotone mapping, split equality, strong convergence, uniform continuity.

Category: **Analysis**

****

5 BIOSTATISTICS

The link between exchange rates and commodity prices is not always the same: Lessons from Mozambique

Alberto Mulenga[a] , **Marta Faias**[b] , **Pedro Mota**[b] & **Joaquim Pina**[b]
[a] Universidade Eduarddo Mondlane
alberto.mulenga@uem.ac.mz

[b] Universidade NOVA de Lisboa - Portugal
mcm@fct.unl.pt , pjpm@fct.unl.pt and pagl@fct.unl.pt

Abstract

We estimate multivariate autoregressive conditional heteroskedasticity models (M-GARCH) seeking to identify the patterns of co-movement among exchange rates and commodity prices, in different situations as to marked behavior in terms of currency appreciation, depreciation and stability. We derive lessons from Mozambique, particularly tailored to the aimed setup and further adding up a perviewed Dutch disease situation. Taking data for Mozambican New Metical (MZN) against South African Rand (ZAR) and also against Great Britain Pound (GBP), plus the quote of price of Coal for Africa (CZA), we employ models in the class of conditional correlations of the M-GARCH family and run three-variate models to a full sample from 2010-2014 and to four sub-samples: 2010, time of depreciation; 2011, time of appreciation; 2012, stability period; 2013-2014, stability with some appreciation. Results suggest that the link across markets, namely exchange rates and commodity prices, is not always the same, it depends on the type of "systematic" behavior of the exchange rate.

Keywords: Exchange rates, commodities, M-GARCH, transmission across markets.

Category: **Statistics**

References

[1] Bauwens,L., Laurent, S. and Rombouts,J.V.K.(2006). Multivariate GARCH models: A Survey. *Journal of Applied Econometrics*, 21, pp. 79 - 109. DOI:10.1002/jae.842.

[2] Bollerslev,T.(1990). Modeling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model. *Review of Economics and Statistics*, 72, pp. 498 - 505. DOI:10.2307/2109358.

[3] Bollerslev,T. Engle,R.F. and Wooldridge,J.M.(1988). A capital asset pricing model with time varying covariances. *Journal of Political Economy*, 96, pp. 116 - 131. DOI:10.1086/261527.

[4] Engle, R.F.(2002). Dynamic conditional correlation - a simple class of multivariate GARCH models. *Journal of Business and Economic Statistics*, 20, pp. 339 - 350. DOI: 10.1198/073500102288618487

****

Effectiveness of vaginal microbicides in preventing HIV transmission

Alfred Musekiwa[a], **Nimasha B. Fernando**[b] & **Samuel A. Abariga**[c]
[a] School of Health Systems and Public Health, Faculty of Health Sciences, University of Pretoria, Pretoria, South Africa
Alfred.Musekiwa@up.ac.za

[b] Department of Epidemiology and Public Health, University of Maryland School of Medicine, Baltimore, MD, USA
nbfernando7@gmail.com

[c] Department of Epidemiology, Johns Hopkins Bloomberg School of Public Health, Baltimore, MD, USA
abarigas@gmail.com

Abstract

We evaluated evidence on the effectiveness of vaginal microbicides in preventing HIV transmission in women. We carried out a systematic review through a comprehensive search of relevant electronic databases for eligible randomised controlled trials (RCTs) published through June 2019. Two authors independently screened titles and abstracts according to eligibility criteria, then extracted data and assessed risk of bias of included studies. We conducted a random-effects meta-analysis of risk ratios (RR) of HIV infection and assessed heterogeneity using the Chi-squared test and the I^2 statistic. Sources of heterogeneity were investigated through subgroup analysis, publication bias was assessed using funnel plots, and certainty of evidence was graded using GRADEPro software. We included 18 RCTs which enrolled 40,048 sexually active, HIV-negative, non-pregnant women, aged 16 years and older, mainly from sub-Saharan Africa. The intravaginal ring containing Dapivirine significantly reduced HIV risk by 29% (RR 0.71, 95% CI: 0.57–0.89; 2 RCTs, 4,564 women, moderate certainty of evidence). Estimates of effect of tenofovir 1% (RR 0.83, 95% CI: 0.65–1.06), nonoxynol-9 (RR 1.15, 95% CI: 0.93–1.42), cellulose sulphate (RR 1.16, 95% CI: 0.61–2.21), SAVVY (RR 1.34, 95% CI: 0.69–2.59), Carraguard (RR 0.89, 95% CI: 0.71–1.10), BufferGel (RR 1.02, 95% CI: 0.71–1.46), 0.5% PRO2000 (RR 0.88, 95% CI: 0.60–1.28) and 2% PRO2000 (RR 0.81, 95% CI: 0.58–1.12) failed to reach statistical significance; each had low certainty of evidence. The long-acting intravaginal ring containing Dapivirine significantly reduced risk of HIV transmission in women by 29%. The remaining microbicides had no evident effect.

Keywords: Dapivirine, HIV prevention, meta-analysis, systematic review, vaginal microbicide

Category: **Biostatistics**

****

Quasi-Stationary Distribution of a Discrete SIQ Stochastic Model

Aubrey Ndovie[a] & **Mokaedi V. Lekgari**[b]
aubrey.ndovie@studentmail.biust.ac.bw

lekgarim@biust.ac.bw

ABSTRACT

We study the quasi-stationary distribution of a discrete SIQ Stochastic model. The model chosen investigated the dynamic behaviour of the recent novel Corona Virus (Covid-19) as proposed by Din et al. (2020). The Quasi-stationary distribution helps us understand the long term behaviour of stochastic processes which persist before absorption. First we define the concept of quasistationarity, we then approximate the quasi-stationary distribution of the model and investigate the resulting simulations of the distribution. From observing the simulations we notice convergence towards stationarity under certain conditions.

Keywords: Quasi-Stationary Distribution, Stochastic Models, Markov Processes,Probability Measures.

References

[1] A. Din, Y. Li,T.Khan and G. Zaman (2020): Mathematical Analysis of the Spread and Control of the Novel Corona Virus in China). *Chaos, Solitons and Fractals, 141*.

****

The Marshall-Olkin-Odd Power Generalized Weibull-G Family of Distributions with Applications of COVID-19 Data

Fastel Chipepa[a], **Thatayaone Moakofi**[b] & **Broderick Oluyede**[c]

[a] Department of Mathematics and Statistical Sciences, Botswana International University of Science and Technology, Palapye

ABSTRACT

Attempts have been made to define new families of distributions that provide more flexibility for modelling data that is skewed in nature. In this work, we propose a new family of distributions called Marshall-Olkin-odd power generalized Weibull (MO-OPGW-G) distribution based on the generator pioneered by Marshall and Olkin [**?**]. This new family of distributions allows for a flexible fit to real data from several fields, such as engineering, hydrology and survival analysis. The mathematical and statistical properties of these distributions are studied and its model parameters are obtained through the maximum likelihood method. We finally demonstrate the effectiveness of these models via simulation experiments and applications to COVID-19 daily deaths data sets.

Keywords: Marshall-Olkin-G, Maximum Likelihood Estimation, Power Generalized Weibull Distribution, Simulation.

****

Comparison of Parametric methods and Matrix completion methods on recovery of missing data on Ecological data : Case of Principal Component Analysis

Gothatamang Patrick Nthoiwa[a], **Katleho D Makatjane**[b] & **Olusegun S Ewemooje**[c] & **Ramasaymy Sivasamy**[D]

[a] Botswana University of Agriculture and Natural Resources , Department Mathematics and Biometry
gpnthoiwa@hotmail.com

[b] University of Botswana , Department of Statistics, Botswana
lMakatjanek@ub.ac.bw

[c] University of Botswana , Department of Statistics, Botswana
EwemoojeOS@ub.ac.bw

[D] University of Botswana , Department of Statistics, Botswana
sivasamyr@ub.ac.bw

Abstract

Standard multivariate techniques like Principal Component analysis(PCA) which are based on the Singular Value Decomposition (SVD) of a data matrix (e.g., covariance matrix) require complete data . Missing data or incomplete data is a relatively common problem in almost all research that involves data collection including ecological data which uses PCA for Ordination that is displaying the differences between samples graphically into fewer dimensions than the original data.In this paper we compare the parametric methods of Multiple imputation and the matrix completion method on the recovery of missing data . The parametric methods are based on assumption of normal, independent distribution while the matrix completion is based on assumptions of low rank approximation .

Keywords: : multiple imputation, EM algorithm, Singular value decomposition, Principal component analysis, Missing Values ;

Category: **Statistics**

References

[1] Wright, J., Ganesh, A., Rao, S. R., Peng, Y., & Ma, Y. (2009). Robust Principal Component Analysis: Exact Recovery of Corrupted Low-Rank Matrices via Convex Optimization.l. *Retrieved 5 9, 2022, from http://papers.nips.cc/paper/3704-robust-principal-component-analysis-exact-recovery-of-corrupted-low-rank-matrices-via-convex-optimization.pdf*

[2] Serneels, S., & Verdonck, T. (2008). Principal component regression for data containing outliers and missing elements.. *Computational Statistics & Data Analysis, 52(11), 1712-1727.*

[3] hang, F., Liu, Y., Cheng, J., & Cheng, H. (2014). Robust Principal Component Analysis with Missing Data.s. *CIKM '14: Proceedings of the 23rd ACM International Conference on Conference on Information and Knowledge ManagementNovember 2014 Pages 1149–1158.*

[4] osse, J., Pagès, J., & Husson, F. (2011). Multiple imputation in principal component analysis. *Advanced Data Analysis and Classification, 5(3), 231-246.*

****

Using Generalized Structured Additive Regression Models to determine factors associated with and clusters for COVID-19 hospital deaths in South Africa

Innocent Maposa
Witwatersrand University, School of Public, Division of Epidemiology and Biostatistics & 27 St Andrews, Park Town, Faculty of Health Sciences
Innocent.Maposa@wits.ac.za

Abstract

Background: The first case of COVID-19 in South Africa was reported in March 2020 and the country has since recorded over 3.6 million laboratory-confirmed cases and 100 000 deaths as of March 2022. Transmission and infection of SARS-CoV-2 virus and deaths in general due to COVID-19 have been shown to be spatially associated but spatial patterns in in-hospital deaths have not fully been investigated in South Africa. This study uses national COVID-19 hospitalization data to investigate the spatial effects on hospital deaths after adjusting for known mortality risk factors.
Methods: COVID-19 hospitalization data and deaths were obtained from the National Institute for Communicable Diseases (NICD). Generalized structured additive logistic regression model was used to assess spatial effects on COVID-19 in-hospital deaths adjusting for demographic and clinical covariates. Continuous covariates were modelled by assuming second-order random walk priors, while spatial autocorrelation was specified with Markov random field prior and fixed effects with vague priors respectively. The inference was fully Bayesian.
Results: The risk of COVID-19 in-hospital mortality increased with patient age, with admission to intensive care unit (ICU) (aOR=4.16; 95% Credible Interval: 4.05-4.27), being on oxygen (aOR=1.49; 95% Credible Interval: 1.46-1.51) and on invasive mechanical ventilation (aOR=3.74; 95% Credible Interval: 3.61-3.87). Being admitted in a public hospital (aOR= 3.16; 95% Credible Interval: 3.10-3.21) was also significantly associated with mortality. Risk of in-hospital deaths increased in months following a surge in infections and dropped after months of successive low infections highlighting crest and troughs lagging the epidemic curve. After controlling for these factors, districts such as Vhembe, Capricorn and Mopani in Limpopo province, and Buffalo City, O.R. Tambo, Joe Gqabi and Chris Hani in Eastern Cape province remained with significantly higher odds of COVID-19 hospital deaths suggesting possible health systems challenges in those districts.
Conclusion: The results show substantial COVID-19 in-hospital mortality variation across the 52 districts. Our analysis provides information that can be important for strengthening health

policies and the public health system for the benefit of the whole South African population. Understanding differences in in-hospital COVID-19 mortality across space could guide interventions to achieve better health outcomes in affected districts.

Keywords: COVID-19; spatial effects; health systems; hospitalizations; nonlinear effects; clusters; deaths

Category: **Statistics**

References

[1] Nikolaus . Umlauf et al. Structured additive regression models: An R interface to Bayes X *J Stat Softw*, 63(21):1–46, 2001.

****

A Multivariate Discrete Time-to-Event Model for Multiple Recurring Events

Jesca M. Batidzirai[a], **Samuel O.M. Manda**[b] & **Henry G. Mwambi**[c]

[a] University of KwaZulu-Natal
batidzirai@ukzn.ac.za

[b] University of Pretoria
samuel.manda@up.ac.za

[c] University of KwaZulu-Natal
mwambi@ukzn.ac.za

Abstract

Recent developments in multi-state models have often considered discrete time when modeling transition intensities. These developments have included univariate multilevel models to account for possible dependence among events that are recurrent in the same subject. We propose a multivariate discrete-time survival model with multiple state transitions where each specific transition has its own separate random effect and the interest is on measuring dependence both between-and within-transitions. Multivariate Normal model for the random effects is suggested. The model parameters are estimated using maximum likelihood methods with non-adaptive Gaussian quadratures numerical integration. The proposed methodology was applied to a real-life data set from an ongoing longitudinal study based on life course history on marriage formation and dissolution events in rural KwaZulu-Natal of South Africa. The five events under consideration in the multi-state process were transition into a first marriage, exiting a marriage through separation, exiting a marriage through death of a partner, re-marriage after a separation and remarriage from widowhood. The model produced smaller standard errors and narrower confidence intervals compared to those from univariate models. Results showed the presence of very small unobserved subject-to-subject heterogeneity for each transition and a weak positive correlation between events.

Keywords: Discrete-time-to-event, Multi-state models, Random effects, Multivariate Competing risks, Gauss-Hermite

Category: **Biostatistics**

References

[1] Tutz, G., and Schmid, M. (2016). Modeling discrete time-to-event data. *New York: Springer*.

[2] Schmid, M., and Berger, M. (2021). Competing risks analysis for discrete time-to-event data.*Wiley Interdisciplinary Reviews: Computational Statistics, 13(5), e1529*.

****

Investigating currency and stock exchange risk within South African Financial markets using the GPD-Normal-GPD Extreme Value Mixture Model

Knowledge Chinhamu[a], **Retius Chifurira**[b] & **Kimera Naradh**[c]

[a]School of Mathematics,Statistics and Computer Science, University of KwaZulu-Natal, South Africa
chinhamu@ukzn.ac.za

[b] School of Mathematics,Statistics and Computer Science, University of KwaZulu-Natal, South Africa
chifurira@ukzn.ac.za

[c]School of Mathematics,Statistics and Computer Science, University of KwaZulu-Natal, South Africa
kimmi.naradh@gmail.com

Abstract

The onset of the COVID-19 pandemic has decimated several industries within the South African economy including the financial industry. The impact of the COVID-19 crisis on economic activity, commercial banks and financial stock markets required the central bank to have tactical responses and risk management tools that ensure financial stability. This study aims to investigate the fit of the GPD-Normal-GPD (GNG) mixed models fitted for three FTSE/JSE indices namely All Share Index (ALSI), Banks Index and Mining Index and the USD/ZAR currency exchange rate. Value at Risk (VaR) assesses market risk and many financial corporations often aspire reliable VaR estimates. VaR estimates and the Kupiec likelihood backtesting procedure are calculated to evaluate the tail behaviour of the fitted GNG models. Favourable results highlight the robustness of the GNG mixed model for each daily returns. This research confirms the use of the GNG mixed model aimed at accounting for the many characteristics of financial data such as heavy tails and asymmetry. Financial practitioners looking to curb losses and explore alternatives for financial modelling in the South African financial industry using an extreme value mixed model approach may gain the most by implementing the GNG model.

Keywords: GNG,Kupiec likelihood ratio test, Value at Risk

Category: **Statistics**

References

[1] Coles, S. (2001). An introduction to statistical modeling of extreme values. Springer Series in Statistics. Springer-Verlag, London.

[2] Kupiec, P. (1995). Techniques for verifying the accuracy of risk measurement models. The Journal of Derivatives, 3(2), 73-84

[3] Naradh, K., Chinhamu, K. and Chifurira, R. (2021). Estimating the value-at-risk of JSE indices and South African exchange rate with Generalized Pareto and stable distributions. Investment Management and Financial Innovations, 18(3), 151-165

[4] Zhao, X., Scarrott, C., Oxley, L., and Reale, M. (2014). Extreme value modelling for forecasting market crisis impacts. In The Global Financial Crisis . Routledge.

****

GIS based spatial modelling of Covid-19 incidence rates in Mozambique

Manuel José Bila[a], **Mário Chelengo**[b], **Osvaldo Loquiha**[c], & **Sansão A. Pedro**[d]

[a] Department of Mathematics and Informatics, Eduardo Mondlane University, 257, Maputo, Mozambique
manueljosebila7@gmail.com

[b] Department of Mathematics and Informatics, Eduardo Mondlane University, 257, Maputo, Mozambique
mariochelengo@gmail.com

[c] Department of Mathematics and Informatics, Eduardo Mondlane University, 257, Maputo, Mozambique
osvaldo.loquiha@uem.ac.mz

[c] Department of Mathematics and Informatics, Eduardo Mondlane University, 257, Maputo, Mozambique
sansao.pedro@uem.ac.mz

Abstract

The World Health Organization declared COVID-19 pandemic outbreak a global concern, due to the rapid spread of the disease and the United Nations described it as a social, human and economic crisis. In recent years, studies about COVID-19 pointed specific risk factors as agents that increase the risk of disease or infection. Hence, the present study aims to identify and determine influence of potential risk factors contributing to the COVID-19 incidence rates, considering different spatiotemporal variation in Mozambique. Spatial regression models were

applied in this study to fulfill the objective. Data from demographic, educational, environmental and socioeconomic thematic were collected and analyzed to explain the spatial and temporal variability of this disease incidence. The developed models are grouped in global (Ordinary Least Squares, Spatial Lag Model and Spatial Error Model) and local regression models (Geographic Weighted Regression and Multi-scale Geographic Weighted Regression). As results, the models identified, at province level, the following variables as risk factors: Literacy level and Health access points rates in the first wave; Malaria, Education, Literacy and HIV rates in second wave; and HIV and doctors per health access points units rates in third wave, with R2 greater than 79%. It take us to conclude that interventions in Literacy Level and HIV rates are globally required. At district level, the models pointed Education Access Points rate with R2 values of 78.1%, 74.6% and 78.4% in first, second and third waves respectively, as risk factor. Therefore, policies related to Education Access Points locally in the south and south side of center region of Mozambique.

Keywords: Pandemic, Risk Factors, Spatiotemporal Variation, Spatial Regression Models, Education Access points.

Category: **other**

****

Microscopic Numerical Simulations of Stochastic Epidemic Models

Mokaedi V. Lekgari
Botswana International University of Science and Technology
lekgarim@biust.ac.bw

Abstract

We study epidemic models with interest in probabilistic individuals' interactions as a driver of state variable dynamics in the spread of communicable diseases. We implement numerical simulations for discrete stochastic processes such as branching processes, discrete-time Markov chains (DTMCs) and continuous-time Markov chains (CTMCs) and continuous stochastic processes such as stochastic differential equations (SDEs) systems.

Keywords: Epidemic models; Simulation; Stochastic Models

****

Topp-leone gompertz exponentiated half logistic-g (tl-gom-ehl-g) family of distributions with applications

Neo Dingalo[a], **Broderick Oluyede**[b], & **Fastel Chipepa**[d]
[a] Botswana International University of Sciences and Technology, Botswana
neo.dingalo@studentmail.ac.bw

[b] Botswana International University of Sciences and Technology, Botswana
oluyedeo@biust.ac.bw

[c] Botswana International University of Sciences and Technology, Botswana
chipepaf@biust.ac.bw

Abstract

This paper introduces and investigates a new family of distributions called the Topp-Leone-Gompertz-exponentiated half logistic-G (TL-Gom-EHL-G) distribution. Some mathematical and statistical properties of this family of distributions are derived. To estimate and evaluate the model parameters, the maximum likelihood estimation technique is used, and the consistency of maximum likelihood estimators is examined using simulation. Applications to two real data sets from different areas demonstrates the usefulness and versatility of the TL-Gom-EHL-G family of distributions.

Keywords: Topp-Leone Distribution, Gompertz Distribution, Exponentiated Half Logistic Distribution, Maximum Likelihood Estimation, Simulation Study, Goodness-of-fit Statistics

Category: **Statistics**

References

[1] Gompertz, B. (1825). On the Nature of the Function Expressive of the Law of Human Mortality, and on a New Mode of Determining the Value of Life Contingencies. Philosophical Transactions of the Royal Society of London, 115, 513–583.

[2] Oluyede, B., Chamunorwa, S., Chipepa, F. and Alizadeh, M. (2022). The Topp-Leone Gompertz-G Family of Distributions with Applications. Journal of Statistics and Management Systems, 1-25.

****

Statistical education in Mozambique: what lessons we learned and what's ahead

Osvaldo Loquiha
Department of Mathematics and Informatics, Faculty of Sciences, Universidade Eduardo Mondlane
osvaldo.loquiha@gmail.com

Abstract

Since 2004, high education in statistics is offered in Mozambique with over 350 graduates now called upon to work in various spheres of economic and social development, such as the institutions of the national statistical system, banking and finance, insurance companies, educational and research institutions, industry, among others. With the advancements in the computing capabilities and ever-increasing demand in data analysis, statistics can no longer be seen as just another mathematical area but rather as the science of learning from data, with a large-scale use of mathematical and computational tools and a focused scientific agenda. I will try to reflect on what have we learnt over the last 18 years and what opportunities lay ahead amidst the growth of data science and artificial intelligence.

Keywords: statistical education, statistics, data science, artificial intelligence

Category: **Statistics**

****

Cryptocurrencies and tokens lifetime analysis from 2009 to 2021

Paul Gatabazi[a,b,c], **Gaetan Kabera**[d], **Jules Clement Mba**[a], **Edson Pindzae**[e] & **Sileshi Fanta Melessef**[f]

[a] University of Johannesburg, South Africa
[b] University of Rwanda, Rwanda
[c] University of the Witwatersrand, South Africa
[d] University of South Africa, South Africa
[e] University of Pretoria, South Africa
[f] University of KwaZulu Natal, South Africa
gatabazip@gmail.com

Abstract

The success of Bitcoin has spurred emergence of countless alternative coins with some of them shutting down only few weeks after their inception, thus disappearing with millions of dollars collected from enthusiast investors through initial coin offering (ICO) process. This has led investors from the general population to the institutional ones, to become skeptical in venturing in the cryptocurrency market, adding to its highly volatile characteristic. It is then of vital interest to investigate the life span of available coins and tokens, and to evaluate their level of survivability. This will make investors more knowledgeable and hence build their confidence in hazarding in the cryptocurrency market. Survival analysis approach is well suited to provide the needed information. In this study, we discuss the survival outcomes of coins and tokens from the first release of a cryptocurrency in 2009. Non-parametric methods of time to event analysis namely Aalen Additive Hazards Model (AAHM) trough counting and martingale processes, Cox Proportional Hazard Model (CPHM) are based on six covariates of interest. Proportional hazards assumption (PHA) is checked by assessing the Kaplan-Meier estimates of survival functions at the levels of each covariate. The results in different regression models display significant and non-significant covariates, relative risks and standard errors.Among the results, it was found that cryptocurrencies under standalone blockchain were at a relatively higher risk of collapsing. It was also found that the 2013-2017 cryptocurrencies release was at a high risk as compared to 2009-2013 release and that cryptocurrencies for which headquarters are known had the relatively better survival outcomes. This provides clear indicators to watch out for while selecting the coins or tokens in which to invest.

Keywords: Cryptocurrency, blockchain, survival function, risk, weight, hazard ratio

Category: **Statistics**

****

Estimating the risk of SARS-CoV-2 deaths using a Markov Switching-GARCH-type model with heavy-tailed distributions for South Africa

Retius Chifurira[a], **Knowledge Chinhamu**[b] & **Nobuhle Mthethwa**[c]
[a]School of Mathematics,Statistics and Computer Science, University of KwaZulu-Natal, South Africa
chifurira@ukzn.ac.za

[b] School of Mathematics,Statistics and Computer Science, University of KwaZulu-Natal, South Africa
chinhamu@ukzn.ac.za

[c]School of Mathematics,Statistics and Computer Science, University of KwaZulu-Natal, South Africa
boohler88@gmail.com

Abstract

Research purpose: SARS-CoV-2 (Covid-19 virus) infection exposed the unpreparedness of African countries to health-related issues, South Africa included. Africa recorded more than 211 853 deaths due to Covid-19. South Africa faced the highest number of casualties. This study aimed to estimate the risk of fatalities due to Covid-19 infection for an African country hard hit by the pandemic.
Research approach/ Methodology: We use the value-at-risk (VaR) concept in financial time series analysis to estimate the risk of deaths. We used the data set of the daily number of deaths due to Covid-19 infection in this study. The data exhibits non-normality, structural breaks and volatility clustering. The Markov switching-GARCH-type (MS-GARCH) model combined with heavy-tailed distributions is appropriate for this data, the heavy-tailed distributions employed are the Student's-t distribution (StD), Skewed Student's-t distribution (SStD), normal reciprocal inverse Gaussian distribution (NRIGD), and Pearson's type IV distribution (PIVD), were developed to estimate the risk of deaths. The model performances were assessed using the Kupiec likelihood ratio test.
Main findings: The results show that the MS-GARCH-NRIGD is the best model at 90 percent. At 95 percent the MS-GARCH-SStD and MS-GARCH-PIVD are the best models, and MS-GARCH-StD, MS-GARCH-NRIGD, MS-GARCH-PIVD are the best at 97.5 percent. The MS-GARCH-NRIG is the best model at 99 percent. Overall MS-GARCH-NRIGD is the best model, it outperforms all the models at almost all the levels. The VaR results showed that with a probability of 0.95, the number of deaths that will occur on the 27th, 28th and 29th of August 2021 will be greater than 278,211, and 134 respectively, the actual values are 361,274 and 134.
Practical implications: The outcomes of this study are expected to be of salient value to health analysts, policymakers, and researchers in bio-statistics, thus giving a better understanding of the Covid-19 pandemic.
Contributions/value-add: The Markov-switching (3)-GARCH model combined with heavy-tailed distributions provides a good option for modelling the daily number of deaths in a pandemic.

Keywords: MS-GARCH, SARS-CoV-2 infection, death, volatility models; value-at-risk; heavy-tailed distributions; back testing.

Category: **Statistics**

References

[1] Abdool Karim, S.S. and de Oliveira, T., (2021). New SARS-CoV-2 variants—clinical, public health, and vaccine implications. New England Journal of Medicine, 384(19), pp.1866-1868.

[2] Kupiec, P. (1995). Techniques for verifying the accuracy of risk measurement models. The Journal of Derivatives, 3(2), 73-84

[3] Naradh, K., Chinhamu, K. and Chifurira, R. (2021). Estimating the value-at-risk of JSE indices and South African exchange rate with Generalized Pareto and stable distributions. Investment Management and Financial Innovations, 18(3), 151-165

[4] Ardia, D., Bluteau, K., Boudt, K. and Catania, L., (2018). Forecasting risk with Markov-switching GARCH models: A large-scale performance study. International Journal of Forecasting, 34(4), pp.733-747.

****

A multivariate spatial autocorrelation in cluster detection

Timotheus Brian Darikwa[a], **Samuel Manda**[b] & **M'aseka Lesaoana**[a]

[a] University of Limpopo, South Africa
timotheus.darikwa@ul.ac.za

[b] University of Pretoria, South Africa
samuel.manda@up.ac.za

[a] University of Limpopo, South Africa
lesaoana.maseka@ul.ac.za

Abstract

This paper is concerned with the development and application of a new measure of multivariate spatial autocorrelation. The new measure is based on the approach of canonical correlation. Our derived statistic provided an accurate degree of multivariate spatial clustering and identified joint local spatial clusters of the analysed mortality rates. Joint local clusters were revealed for the three mortality rates. The identified co-clusters may be targeted in integrated intervention and monitoring programs. The usefulness of the proposed multivariate spatial clustering measure was assessed by a comparative study with another recently developed multivariate spatial autocorrelation measure and analysis of noncommunicable disease-related mortality rates in South Africa.

Keywords: Multivariate analysis, Spatial statistics, Moran's I, Canonical correlation , Regression analysis

Category: **Statistics**

References

[1] Wartenberg,D. (1985): Multivariate spatial correlation: A method for exploratory geographical analysis. *Geographical Analysis 2, 263 - 283*.

[2] Anselin, L., Syabri, I. and Smirnov, O. (2002):Visualizing multivariate spatial correlation with dynamically linked windows. *Paper presented at: New Tools for Spatial Data Analysis 2002. Proceedings of the Specialist Meeting; Santa Barbara 2002 July 8–10; Illinois, Unite States of America*.

[3] Lee, S. I. (2001):Developing a bivariate spatial association measure: An integration of Pearson's r and Moran's I. *Journal of Geographical Systems 3, 369 - 385*.

****

6 Data Science

On quantum Fourier transform and quantum phase estimation

Adamu Salawu[a], & **Ezekiel Ayoola**[b]

[a] Department of Mathematical Sciences, Faculty of Natural Sciences, Prince Abubakar Audu University, Anyigba, Kogi State.
saladinoyi@gmail.com

[b]Department of Mathematics, Faculty of Science, University of Ibadan, Oyo State.
eoayoola@gmail.com

Abstract

In this paper, a detailed review of the concepts of Quantum Fourier Transformation (QFT) and Quantum Phase Estimation (QPE) is given. We present the mplementation of these concepts using Qiskit simulation in which QFT is performed on qubits encoded with real values. In addition, we exmeplify an instance of QPE in estimating a parameter to a level of four qubits precision. Findings indicate that these concepts are simply implementable as against the theoretical descriptions and in addition, QPE as a subroutine or black box within quantum algorithms is a useful tool in approximating unknown phase factor of simulation algorithms. Lastly, various promising applications of QFT and QPE are stated.

Keywords: Quantum computing, quantum Fourier transform, quantum circuit, quantum algorithms, quantum measurement.

Category: **Computational Mathematics**

****

Learning deep linear neural networks: Riemannian gradient flows and convergence to global minimizers

Bubacarr Bah[a], **Holger Rauhut**[b], **Ulrich Terstiege**[b] & **Michael Westdickenberg**[c]
[a] African Institute for Mathematical Sciences & Stellenbosch University, South Africa
bubacarr@aims.ac.za

[b] Chair for Mathematics of Information Processing, RWTH University, Germany
rauhut@mathc.rwth-aachen.de, terstiege@mathc.rwth-aachen.de

[c] Institute for Mathematics, RWTH University, Germany
mwest@instmath.rwth-aachen.de

Abstract

We study the convergence of gradient flows related to learning deep linear neural networks (where the activation function is the identity map) from data. In this case, the composition of the network layers amounts to simply multiplying the weight matrices of all layers together, resulting in an overparameterized problem. The gradient flow with respect to these factors can be re-interpreted as a Riemannian gradient flow on the manifold of rank-r matrices endowed with a suitable Riemannian metric. We show that the flow always converges to a critical point of the underlying functional. Moreover, we establish that, for almost all initializations, the flow converges to a global minimum on the manifold of rank matrices for some $k \leq r$.

Keywords: Gradient flows, Deep linear neural networks, Manifold of rank-r matrices, Global minimum, Overparameterized problem, Riemannian gradient flow

Category: **Data Science**

References

[1] Bah, B., Rauhut, H., Terstiege, U., Westdickenberg, M. (2022). Learning deep linear neural networks: Riemannian gradient flows and convergence to global minimizers. *Information and Inference: A Journal of the IMA, 11(1), 307-353*.

[2] Chitour, Y., Liao, Z., Couillet, R. (2018). A geometric approach of gradient descent algorithms in neural networks. *arXiv preprint arXiv:1811.03568*.

[3] Arora, S., Cohen, N., Golowich, N., Hu, W. (2018). A convergence analysis of gradient descent for deep linear neural networks. *arXiv preprint arXiv:1810.02281*.

****

Sharp maximal inequalities for Bessel processes

Cloud Makasu
University of the Western Cape, Private Bag X17, Cape Town, South Africa
cmakasu@uwc.ac.za

Abstract

We prove a one-sided sharp maximal inequality for a randomly stopped Bessel process of dimension $1 \leq \alpha < 2$. For the special case when $\alpha = 1$, we obtain a sharp Burkholder-Gundy inequality for Brownian motion as a consequence.

Keywords: Bessel processes, Burkholder-Gundy inequality, optimal stopping problem

References

[1] Abundo, M. (2017): *The mean of the running maximum of an integrated Gauss-Markov process and the connection with its first-passage time.* Stochastic Anal. Appl. **35**, 499–510.

[2] Burkholder, D. L. (1977): *Exit times of Brownian motion, harmonic majorization, and Hardy spaces.* Adv. in Math. **26**, 182–205.

[3] Dante DeBlassie, R. (1987): *Stopping times of Bessel processes.* Ann. Probab. **15**, 1044–1051.

[4] Dubins, L. E. and Schwarz, G. (1988): *A sharp inequality for submartingales and stopping times.* Astérisque **157-158**, 129–145.

****

Convergence of gradient descent to the global minimum for learning deep linear neural networks

Gabin Maxime Nguegnang[a], **Holger Rauhut**[b] & **Ulrich Terstiege**[c]

[a] RWTH Aachen University, Germany
nguegnang@mathc.rwth-aachen.de

[b] RWTH Aachen University, Germany
rauhut@mathc.rwth-aachen.de

[c] RWTH Aachen University, Germany
terstiege@mathc.rwth-aachen.de

Abstract

In this work, we study the convergence properties of gradient descent for training deep linear neural networks, i.e., deep matrix factorizations, by extending a previous analysis for the related gradient flow. We show that under suitable conditions on the step sizes gradient descent converges to a critical point of the loss function, i.e., the square loss in this study. Furthermore, we demonstrate that for almost all initializations gradient descent converges to a global minimum in the case of two layers. In the case of three or more layers we show that gradient descent converges to a global minimum on the manifold matrices of some fixed rank, where the rank cannot be determined a priori.

Keywords: Deep Learning, Gradient descent, Boundedness, Balancedness, Convergence

Category: **Data Science**

References

[1] Nguegnang, G.M., Rauhut, H. and Terstiege, U. (2021): Convergence of gradient descent for learning linear neural networks. *arXiv preprint arXiv:2108.02040*

[2] Bah, B., Rauhut, H., Terstiege, U. and Westdickenberg, M. (2021). Learning deep linear neural networks: Riemannian gradient flows and convergence to global minimizers. *Information and Inference: A Journal of the IMA 11(1), 307 - 353*.

****

Coping mechanism of spirituality in times of crisis: A sentiment analysis of tweets data

Jules Clement Mba[a] & **Mdu Biyase**[b]
[a] University of Johannesburg, South Africa
jmba@uj.ac.za
[b] University of Johannesburg, South Africa
mbiyase@uj.ac.za

Abstract

Covid-19 pandemic has brought devastating impact in the world. The surge in the number of daily new cases and deaths around the world and in South Africa in particular has increased fear, psychological break-down and uncertainty among the population, leading many to resolve to prayer, meditation, consumption of religious media as coping measures. How have these measures been perceived at each individual level? Social media appears to be a good platform to provide us with significant insights to this question. There has been a growing interest in exploring big data via online platforms which provide real-time evidence to assess social attitudes on various topical issues and saves a lot of time and cost in gathering data. Giving the concise language used in tweets and the ability to obtain information about a public opinion by analyzing Twitter data and automatically classifying their sentiment polarity, twitter sentiment analysis has brought significant insight in many research areas. In this study, the Valence Aware Dictionary and sEntiment Reasoner (VADER) is used to determine the polarity of tweets and to classify them according to multiclass sentiment analysis. We extract four sets of tweets related to Covid-19, religion, life purpose and life experience respectively. A religiosity index is build based on VADER sentiment score which provide a time-varying attitude/sentiment of the population during the crisis period. Then a Granger causality is conducted to test the causal relationship between the four variables. While the findings reveal that religiosity index Granger-causes Life experience, Covid-19 similarly Granger-causes Life experience confirming the reality of perturbations and shocks in the life of many. This study goes further to assess the level of impact each of these variables contribute to individual life at the time of extreme stress and psychological break-down by computing the Conditional Threshold of Depression (CToD). It measures the threshold of the population as a whole entering into depression given that one of the indicator variables is at its threshold of depression. Life purpose sentiment indicator is

top-ranked in terms of Threshold of depression, followed by Life experience sentiment indicator, and therefore these scores can be used to monitor when to place a region on a high alert of depression. Despite the coping mechanism of spirituality or religion, in time of crisis, for the population under study, a Threshold of depression exceeding 0.65 for the religiosity index should be of concern and necessitate complementary intervention.

Keywords: religiosity index, VADER sentiment score, Covid-19

Category: **Data Science**

References

[1] Gilbert, C. J. H. E. (2016): Vader: A parsimonious rule-based model for sentiment analysis of social media text. *in Eighth International Conference on Weblogs and Social Media (ICWSM-14)*. Available at (20/04/16) http://comp.social.gatech.edu/papers/icwsm14.vader.hutto.pdf.

[2] Granger, J. (1969): Investigating causal relations by econometric models and cross-spectral methods. *Acta Physica Polonica B*, vol. 37, pp. 424–438.

[3] Kharde, V. and Sonawane, P. (2016): Sentiment analysis of twitter data: a survey of techniques. *arXiv Prepr. arXiv1601.06971*.

[4] Mestre-Mestre, E.M. (2021): Emotion and Sentiment Polarity in Parliamentary Debate: A Pragmatic Comparative Study. *Corpus Pragmatics* 5, 359–377.

[5] Morgan, J.P. (1996): RiskMetrics Technical Document. *Retrieved October 13, 2013, from http://www.msci.com/resources/research/technical_documentation/td4e.pdf*

[6] Selvaperumal, P. and Suruliandi, D. A. (2014): A Short Message Classification Algorithm for Tweet Classification. *Int. Conf. Recent Trends Inf. Technol.*, pp. 1–3.

[7] Singh, T. and Kumari, M. (2016): Role of Text Pre-processing in Twitter Sentiment Analysis. *Procedia Comput. Sci.*, vol. 89, pp. 549–554.

****

Deep Learning for Partial Differential Equations: Insights and Challenges for extension to free boundary problems

Ronald Katende[a,b,c], **Henry Kasumba**[b], **Godwin Kakuba**[b], **John M. Mango**[b], & **Anna Oleynik**[c]

[a] Kabale University, [b] Makerere University, [c] University of Bergen
rkatende92@gmail.com

Abstract

The study of solutions for Partial Differential Equations (PDEs) has been around for some time and only continues to grow. Various numerical algorithms have been put forth for this matter, the most recent one being machine learning approaches, which enable solution search with less tasking on the person doing the search, due to unsupervised learning. Consider the differential operators as depicted below;

$$\begin{cases} \mathcal{N}(x,u) = 0 \text{ in } \Omega \\ \mathcal{B}(x,u) = 0 \text{ in } \partial\Omega \end{cases} \tag{8}$$

where Ω is the boundary and $\partial\Omega$ is the interior of the boundary. In our work, we briefly review some seminal results in this area, demonstrate how machine learning (deep neural networks (DNNs)) can optimally solve any PDE of this nature, design a suitable loss function, optimal weights selection, contemplate on mathematical questions surrounding deep learning for PDEs such as accuracy, stability and robustness, as well as present results for extension of Deep learning to solving free boundary problems.

Keywords: Partial Differential Equations, Deep Learning, Optimization, Neural Networks, Free Boundary Problems

Category: **Numerical Analysis, Data Science, Partial Differential Equations**

References

[1] M. Raissi and P. Perdikaris and G.E. Karniadakis (2018): Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. *Journal of Computational Physics, 378, 686-707 , 2019, 0021-9991, https://doi.org/10.1016/j.jcp.2018.10.045*

[2] Sifan Wang and Paris Perdikaris (2021), Deep learning of free boundary and Stefan problems. *Journal fo Computational Physics, 10.1016/j.jcp.2020.109914, https://doi.org/10.10162Fj.jcp.2020.109914, Elsevier BV, 428, 109914*

****

Friend of a friend models of network growth

Watson Levens[a,b], **David Sumpter**[b] & **Alex Szorkovszky**[c]

[a] Department of Information Technology, Uppsala University, Sweden

watsonlevens@yahoo.com

[b] Department of Mathematics, University of Dar es Salaam, Tanzania

david.sumpter@it.uu.se

[c] Department of Informatics and RITMO, University of Oslo, Norway

aszorko@gmail.com

Abstract

One of the best-known models in network science is the preferential attachment model, which generates power-law degree distributions. Although this model involves individuals making decisions to form connections (to cite a paper, follow a person on social media, etc.), these decisions are not based solely on local information, i.e. the probability of joining a node depends on a proportion, which is normalized by the sum of the degrees of all members of the population. This is an important limitation in biological, physical, and social interpretations of the model. In this paper, we investigate a truly local model of network formation: based on the idea of a friend of a friend. In this model, individuals joining the network choose one node entirely at random and link to it with probability p, then they choose a neighbour of that node (friend of a friend) and follow that individual with probability q. While such a model has previously been proposed [1] , we identify some important and previously unknown properties of networks that emerge from it. We find that the model can produce power-law degree distributions and node clustering reminiscent of small-world networks. But perhaps most surprisingly, for small p and q = 1, the model produces super-hub networks. Specifically, we prove that for $p = 0$ and $q = 1$, the proportion of non-hubs tends to 1 as the number of nodes goes to infinity. This contradicts claims by Barab´asi [2] and other researchers [1, 3, 4] that this mechanism generates a linear preferential attachment. Instead, we show that power-law degree distributions, small world clustering, and super-hub networks are outcomes of this, more general, yet conceptually simple model.

Keywords: Networks, power-laws, degree distributions, clustering coefficients

References

[1] Jackson MO, Rogers BW. Meeting strangers and friends of friends: How random are social networks?. American Economic Review. 2007 Jun;97(3):890-915.

[2] Barab´asi AL. Network Science. Cambridge, U.K: Cambridge University Press. 2014 [Online]. Available: http://barabasi.com/networksciencebook.

[3] Evans TS, Saram¨aki JP. Scale-free networks from self-organization. Physical Review E. 2005 Aug 31;72(2):026138.

[4] Krapivsky PL, Redner S, Ben-Naim E. A kinetic view of statistical physics. Cambridge University Press; 2010 Nov 18 (p. 454).

****

7 DISCRETE MATHEMATICS

On Uniquely Packable Trees

Alex Alochukwu[a], **Michael Dorfling**[b] & **Elizabeth Jonck**[c] & **Simon Mukwembi**[d]

[a,b,c,d]School of Mathematics, University of the Witwatersrand

[a] alex.alochukwu@wits.ac.za

[c] betsie.jonck@wits.ac.za

[c] simon.mukwembi@wits.ac.za

Abstract

Let G be a connected graph. An i-packing in G is a set of vertices that are pairwise distance more than i apart. A *packing colouring* of G is a partition $X = \{X_1, X_2, \ldots, X_k\}$ of $V(G)$ such that each colour class X_i is an i-packing. The packing chromatic number of G, denoted by $\chi_\rho(G)$ is the minimum order k of a packing colouring. We call a graph G uniquely k-χ_ρ-packable if $\chi_\rho(G) = k$ and G has a unique packing colouring of order k.

In this talk, we present some known results on the packing chromatic number of a graph and investigate uniquely-packable trees. We characterise the uniquely 3-χ_ρ-packable trees, and investigate the existence of trees T for which there is only one packing colouring using $\chi_\rho(T)$ colours.

Keywords: colouring, broadcast, packing, tree, uniquely colourable

Category: **Graph Theory**

****

On proximity and remoteness in directed graphs

Sonwabile Mafunda[a], **Jiangdong Ai**[b], **Stefanie Gerke**[c] & **Gregory Gutin**[d]

[a] University of Johannesburg, South Africa
smafunda@uj.ac.za

[b] Royal Holloway, University of London, United Kingdom
jiangdong.ai2018@live.rhul.ac.uk

[c] Royal Holloway, University of London, United Kingdom
stefanie.gerke@rhul.ac.uk

[d] Royal Holloway, University of London, United Kingdom
g.gutin@rhul.ac.uk

Abstract

In a strong, finite digraph D of order n, the distance $d_D(u, v)$ from vertex u to vertex v is the length of a shortest $u - v$ path in D. The *average distance* $\bar{\sigma}(x)$ of a vertex x of D is the arithmetic mean of the distances from x to all other vertices of D. The *remoteness* $\rho(D)$ and *proximity* $\pi(G)$ of D are the maximum and the minimum of the average distances of the vertices of D, respectively.
In 2021, Ai et al. [1] showed that for any pair of vertices in D, their average distances can differ by no more than $\frac{1}{2}(n-2)$. This suggests a natural question if there is a simple characterisation of all digraphs where all vertices have the same average distance.

In this talk, we discuss the above characterisation for strong tournaments and we further present an infinite family of non-regular strong digraphs D such that $\rho(D) = \pi(D)$.

Keywords: average distance, transmission, distance

Category: **Graph Theory**

References

[1] Ai, J., Gerke, S., Gutin, G. and Mafunda, S. (2021): Proximity and remoteness in directed and undirected graphs. *Discrete Mathematics 344, 112252*.

****

Towards Parallel Implementation of Graph Similarity Algorithms: Identified Issues

Tallman Nkgau[a], **George Anderson**[b]

[a] Department of Computer Science, University of Botswana
nkgautz@ub.ac.bw

[b] Department of Computer Science, University of Botswana
andersong@ub.ac.bw

Abstract

In this research, we consider several graph similarity algorithms, and study issues in parallelising them. We use a High Performance Computing cluster as a target parallel architecture. The algorithms we consider are Jaccard, Neighbour Matching, and Zager-Verghese. A parallel algorithm implementation framework is used to help with this task. Several issues are identified and their mitigation proposed.

Keywords: Graph Similarity, Parallel Algorithms

Category: **Graph Theory**

References

[1] Gonzalez, J.E., Xin, R.S., Dave, A., Crankshaw, D., Franklin, M.J. and Stoica, I (2014): *GraphX: Graph processing in a distributed dataflow framework. In: OSDI, pp. 599–613 (2014)*

[2] Kollias, G., Sathe, M., Schenk, O. and Grama, A (2014): Fast parallel algorithms for graph similarity and matching. *Journal of Parallel and Distributed Computing, vol. 74, no. 5, pp. 2400–2410, 2014, doi: https://doi.org/10.1016/j.jpdc.2013.12.010.*

[3] Zager, L.A. and Verghese, G. C. (2008): Graph similarity scoring and matching. *Applied Mathematics Letters, vol. 21, no. 1, pp. 86–94, Jan. 2008, doi: 10.1016/j.aml.2007.01.006.*

****

8 FINANCIAL MATHEMATICS

Weather derivative pricing under two stochastic volatility model

Betuel Canhanga[a], **Caroline Ogutu**[b]
[a] Department of Mathematics and Informatics - Faculty of Sciences
Universidade Eduardo Mondlane, Maputo - Mozambique,
betuel.canhanga@uem.ac.mz

[b] Department of Mathematics - Faculty of Sciences and Technology
University of Nairobi , Kenya - Nairobi,
cogutu@uonbi.ac.ke

Abstract

One of the recent research interest in finance mathematics is devoted to the weather derivative, a climate financial instrument that is traded depending on the events such as the amount of rainfall, the temperature, amount of windy and others. This instruments can be used to protect investments that for example, depends on a specific weather event. We consider a case where a weather event (for example temperature) is an stochastic process governed by two stochastic volatilities one being more frequently changing than the other. We develop the model, construct an option contract and determine an approximate price of the option.

Keywords: Finance Mathematics, Backward stochastic differential equations, Asymptotic expansion

Category: **Mathematics of Finance**

References

[1] Canhanga, B., Malyarenko, A., Murara, J.-P. and Silvestrov, S. (2016): Pricing European Options under Stochastic Volatilities Models. *Engineering Mathematics I. Electromagnetics, Fluid mechanics, Material physics and financial engineering, Springer, Heidelberg*.

[2] Fouque, J. P., Papaicolaou, G., Sølna, R. S. K., (2011): Multiscale Stochastic Volatility For Equity, Interest Rate, And Credit Derivatives. *Cambridge University Press*

****

Time-consistent investment-proportional reinsurance strategy under a jump-diffusion model

Calisto Guambe
Department of Mathematics and Informatics, Universidade Eduardo Mondlane, Maputo, Mozambique
calisto.guambe@uem.ac.mz

Abstract

In this talk, we formulate a mean-variance portfolio selection problem of an insurer who manages her underlying risk by purchasing proportional reinsurance and investing in a financial market consisting of a bank account and a risky asset following a jump-diffusion dynamics with random parameters. We then obtain a time-consistent equilibrium strategy via a flow of Backward Stochastic Differential Equations. Finally, we apply our results to a mean-reverting Lévy-Ornstein-Uhlenbeck process and obtain closed form solutions.

Keywords: Mean-variance; jump-diffusion; Time consistent problem; BSDEs; Equilibrium strategy; Stochastic interest rate.

Category: **Mathematics of Finance**

****

A Numerical Study of a Weather Derivatives Model

Clarinda Nhangumbe[a,b], **Ercília Sousa**[c] & **Betuel Canhanga**[a]

[a] Universidade Eduardo Mondlane, Departamento de Matemática e Informática

[b] University of Cape Town, Department of Mathematics and Applied Mathematics

clarinda.nhangumbe@uem.ac.mz

[c] CMUC, Department of Mathematics, University of Coimbra, 3001-501 Coimbra, Portugal

Abstract

Weather Derivatives are financial instruments used to cover non-catastrophic weather events and can be expressed in the form of a standard or plain vanilla products, structured or exotics products. The underlying assets in this case are the weather indexes, such as, temperature, rainfall, humidity, wind and snowfall. The complexity of the Weather Derivatives structure exposes the weakness of the Black Scholes framework, therefore, under the risk neutral probability measure the option price of a weather contract can be given as a unique solution of a two dimensional partial differential equation (parabolic in one direction and hyperbolic in other directions), with an initial condition and subjected to adequate boundary conditions. To calculate the price of the option, one can use numerical methods such as the Monte Carlo simulations, and implicit finite difference schemes conjugated with Semi-Lagrangian methods. In this paper is propose two explicit methods namely, first order upwind in hyperbolic direction combined with Lax-Wendroff in parabolic direction and, first order upwind in the hyperbolic direction combined with second order upwind in the parabolic direction. One of the advantages of these methods is the fact that they take in consideration the boundary conditions obtained from the financial interpretation and deal efficiently with the different choices of the convection coefficients.

Keywords: Incomplete markets, numerical methods, partial differential equations, stochastic process, weather derivatives.

Category: **Mathematics of Finance**

****

Local times of deterministic paths and model-free Càdlàg price paths

Darlington Hove[a], **Rafał Łochowski**[b] & **Farai Julius Mhlanga**[c]
[a] Mangosuthu University of Technology, South Africa
hoved@mut.ac.za

[b] Warsaw School of Economics, Poland
rlocho314@gmail.com

[c] University of Limpopo, South Africa
farai.mhlanga@ul.ac.za

Abstract

It appears that typical (model-free) continuous price paths are characterised by many properties attributed in classic probability setting to continuous semimartingales. These include the existence of local times. The objective of this research is to analyse the possible definitions and existence of local times (according to the different definitions) of deterministic model-free price paths. The analysis follows a modification by Łochowski et al. [1] of the scheme introduced by Vovk [2] of a shifted Lesbegue partition $\pi(c, r)$ based on the nets $c \cdot \Re + r = \{c \cdot p + r, p \in Z\}$, $r \in \left(-\frac{c}{2}, \frac{c}{2}\right)$ and c is the interval width per level. The local time for a given level is a measure of how fast the quadratic variation is accumulating for that particular level and is approximated by the product of the interval width and the number of interval crossings for that level [3]. The existence of the local times is analysed for two continuoun but nowhere differentiable functions. One function constructed from the components of the Peano curve and the other by a modified version of the Peano curve. The results show consistent local time estimate for the function constructed from the components of the Peano curve whereas the results from the second function are divergent, suggesting perhaps a need for another normalising factor.

Keywords: local time, interval crossing, nowhere differentiable

Category: **Mathematics of Finance**

References

[1] R. M. Łochowski, J. Obłój, D. K. Prömel, and Siorpaes P. (2021):Local times and tanaka-meyer formulae for cadlag paths. *Electron. J. Probab.*, 26, 1 - 29.

[2] Vovk, V. (2015): Itô calculus without probability in idealized financial markets. *Lith. Math. J.*, 55(2), 270-290.

[3] N. Perkowski and D. J. Prömel. (2015): Local times for typical price paths and pathwise Tanaka formulas. *Electron. J. Probab.*, 20(46), 1–15.

****

Optimal Investment, Consumption and Life Insurance Problem with Stochastic Environments

Elias Offen[a], **Stanley Jere**[b]
[a] University of Botswana
offen@ub.ac.bw
[b] University of Botswana
sjere@mu.edu.zm

Abstract

Optimal investment, consumption and life insurance problem with stochastic environments for a wage-earner is solved in this study. The wage-earner invests in the financial market with one risk-free security, one risky security, receives labor income and has a life insurance policy in the insurance market. A life insurance policy is purchased to hedge the financial wealth for the beneficiary in case of wage earner premature death. The interest rate and the volatility are stochastic. The objective of the wage-earner is to allocate wealth between risky security and risk-free security but also buy a life insurance policy during the investment period to maximize the expected discounted utilities derived from consumption, legacy and terminal wealth over an uncertain lifetime. By applying Bellman's optimality principle, the associated HJB PDE for the value function is established. The power utility function is employed for our analysis to obtain the value function and optimal policies. Finally, numerical examples and simulations are provided.
Keywords: : Investment, life insurance problem, Value function, optimal policies.

Category: **Mathematics of Finance**

References

[1] Bruhn, K., and Steffensen, M. (2011). Household consumption, investment and life insurance. Mathematics and Economics, 48(3), 315-325. http://dx.doi.org/10.2139/ssrn.1586229

[2] Duarte, I., Pinheiro, D., Pinto, A. A., and Pliska, S. R. (2014). Optimal life insurance purchase, consumption and investment on a financial market with multidimensional diffusive terms. Optimization, 63(11), 1737-1760. https://doi.org/10.1080/02331934.2012.665054

****

A Fuzzy Stochastic Model for Pricing Weather Derivatives in Incomplete Markets

Eriyoti Chikodza [a], **Paul Kundai Ziwakaya**[b], **Queensley C. Chukwudum**[c], **Farai Mhlanga** [d],**Elias Mwakilama** [e], **Lungelo Mamba** [f], **Hans Werner Van Wyk** [g]
[a] Department of Mathematics and Computer Science, Great Zimbabwe University, Zimbabwe
echikodza@gzu.ac.zw
[b] Department of Mathematics and Statistical Sciences, Botswana International University of Science and Technology, Botswana

paul.ziwakaya@studentmail.biust.ac.bw

[c] Department of Insurance and Risk Management, University of Uyo, Akwa Ibom State, Nigeria
queensleyv@yahoo.com

[d] Department of Mathematics and Applied Mathematics, University of Limpopo, South Africa
farai.mhlanga@ul.ac.za

[e] Jomo Kenyatta University of Agriculture and Technology, Kenya
mwakilama.elias@students.jkuat.ac.ke

[f] Department of Mathematics, University of Eswatini, Eswatini
lungelomambaa@gmail.com

[g] Auburn University, United States of America
hzv008@auburn.edu

Abstract

Over the years, weather has had a significant impact on the profitability of firms and individuals that trade in energy and commodity markets. For instance, a company that trades in gas used to warm up homes and offices would realise depressed revenues during a warm period, whereas the same firm would make lucrative profits during a cold season. Consequently, energy and commodity market participants have developed mechanisms for managing risks emanating from unfavourable weather conditions. Insurance and weather derivatives are some of the most useful instruments deployed for mitigating losses induced by unexpected weather extremities such as warm temperatures that cause low demand for gas during winter. This paper presents and examines the problem of valuation of temperature derivatives in a fuzzy Ito-Levy context. In our model the evolution of temperature is governed by a geometric Ito-Levy process, in which the jump component describes significant up and down changes in temperature over an infinitesimally short period of time. It is assumed that parameters of the stochastic differential equation cannot be crisply expressed, and for that reason, they are incorporated into the model as membership functions. Using stochastic calculus methodologies coupled with the theory of fuzzy sets, a numerical scheme is proposed for the valuation of temperature derivatives. Ultimately, to gain deeper insight into the scope of the applicability of the pricing model, data from selected African markets will be used for validation purposes.

Keywords: temperature derivative, fuzzy number, fuzzy Ito-Levy process, stochastic differential equation

Category: **Financial Engineering, Computational Intellingence**

References

[1] Lee, C. F., Tseng, G-H., and Wang, S. Y.(2018): A new application of fuzzy set theoty to the Black-Scholes option pricing model. *Expert Systems with Applications.*

[2] Nowak, P.and Romaniuk, M.(2013): A fuzzy approach to option pricing in Levy process setting. *Int. J. Appl. Math., Comput. Sci.*

[3] Spicka, J. and Hnilica, J. M.(2013): A methodical approach to Design and Valuation of weather derivatives in agriculture. *Advances in Meteorology.* Hindawi Publishing Corporation.

****

An optimal portfolio and consumption problem: A viscosity solution approach

Farai Julius Mhlanga[a],
[a] University of Limpopo
farai.mhlanga@ul.ac.za

Abstract

We consider the optimal consumption-investment problem for an investor who trades in a financial market with stochastic coefficients driven by a non-Gaussian Ornstein–Uhlenbeck process. The investor consumes and allocates her wealth in a financial market consisting of a risk-free asset and a risky asset. The objective is to find an allocation process and a consumption pattern which optimises the expected utility. As in [1, 2], we formulate the portfolio optimisation problem as a singular stochastic control problem and solve it using dynamic programming and the theory of viscosity solutions. The value function is characterised as the unique constrained viscosity solution of the corresponding integro-differential variational inequality and explicit results are derived for hyperbolic absolute risk aversion utility functions.

Keywords: optimal investment and consumption, Ornstein-Uhlenbeck process, stochastic volatility model, subordinator, viscosity solutions.

Category: **Mathematics of Finance**

References

[1] Benth, F.E., Karlsen, K.H., and Reikvam, K. (2001). Optimal portfolio management rules in a non-Gaussian market with durability and intertemporal substitution. *Finance and Stochastics 5, 447 - 467*

[2] Delong, L., and Klüppelberg, C. (2008). Optimal investment and consumption in a Black–Scholes market with Lévy-driven stochastic coefficients. *The Annals of Applied Probability*, 18(3), 879 - 908.

****

Stochastic differential game formulation on the reinsurance and investment problem

L. C. Galane*, *University of Limpopo*

This talk presents a stochastic differential game between two insurance companies, a big one and a small one. The distinctions of these companies is that the big company has sufficient asset to invest in a risk-free asset and a risky asset and is allowed to purchase proportional reinsurance or acquire new business, and the small company can transfer part of the risk to a reinsurer via proportional reinsurance. The game studied here is zero-sum, where the big company is trying to maximize the expected exponential utility of the difference between two insurance companies' surpluses at the terminal time to keep its advantage on surplus, while simultaneously the small company is trying to minimize the same quantity to reduce its disadvantage. The relationships between the surplus processes and the price process of the risky asset are considered. The Nash equilibrium strategy is obtained through verification theorem which rest on the stochastic control theory.

References

[1] Mhlanga, F. J., Galane, L. C., Mwareya, N., Chikodza, E., Guambe, C. (2022). Stochastic differential game strategies in the presence of reinsurance and dividend payout. Journal of Industrial and Management Optimization.
[2] Taksar, M., Zeng, X. (2011). Optimal non-proportional reinsurance control and stochastic differential games. Insurance: Mathematics and Economics, 48(1), 64-71.
[3] Zeng, X. (2010). A stochastic differential reinsurance game. Journal of Applied Probability, 47(2), 335-349.
[4] Wang, N., Zhang, N., Jin, Z., Qian, L. (2021). Reinsurance investment game between two mean–variance insurers under model uncertainty. Journal of Computational and Applied Mathematics, 382, 113095.
[5] Li, D., Rong, X., Zhao, H. (2015). Stochastic differential game formulation on the reinsurance and investment problem. International Journal of Control, 88(9), 1861-1877.

****

Application of Generalized Geometric Itô-Lévy Process to Investment-Consumption-Insurance Optimization Problem under Inflation Risk

Obonye Doctor[a]
[a] Botswana International University of Science and Teachnology
doctoro@biust.ac.bw

Abstract

We consider a problem of maximizing the utility of an agent who invests in a stock, money market account and an index bond incorporating life insurance, deterministic income, and consumption. The stock is assumed to be a generalized geometric Itô-Lévy process. Assuming a power utility function, we determine the optimal investment-consumption-insurance strategy under inflation risk for the investor in a jump-diffusion setting using martingale approach.

Keywords: Utility Theory; Portfolio Optimization; Stochastic Control; Itô-Lévy Diffusions; Martingale Method; Life Insurance.

Category: **Mathematics of Finance**

References

[1] Guambe, C. and Kufakunesu, R. (2020): Optimal investment-consumption and life insurance with capital constraints, *Journal of Communications in Statistics-Theory and Methods*, **49**, 648-669.

[2] Kwak, M. and Lim, B. H. (2014): Optimal portfolio selection with life insurance under inflation risk, *Journal of Banking and Finance*, **46**, 59-71.

[3] Øksendal, B. and Sulem, A. (2014): Stochastic Control of Itó-Lévy processes with applications to finance, *Journal of Communications on Stochastic Analysis*, **8**, 1-15.

[4] Zhang, X. and Zheng, X. (2019): Optimal Investment-Reinsurance Policy with Stochastic Interest and Inflation Rates, *Mathematical Problems in Engineering, Hindawi*, **2019**, 1-14.

****

Modelling the Effect of Covid-19 on Employment and GDP: The UK Example

Paul Kundai Ziwakaya [a], **Edward M. Lungu**[b] & **Barbara Szomolay**[c]

[a] Department of Mathematics and Statistical Sciences, Botswana International University of Science and Technology, Private Bag 16, Palapye, Botswana
paul.ziwakaya@studentmail.biust.ac.bw

[b] Department of Mathematics and Statistical Sciences, Botswana International University of Science and Technology, Private Bag 16, Palapye, Botswana
lungue@biust.ac.bw

[c] Systems Immunity Research Institute, Cardiff University of Medicine
SzomolayB@cardiff.ac.uk

Abstract

As the governments prepare for a COVID-19 recovery policy, policymakers must consider the economic consequences of the pandemic on Gross Domestic Product (GDP) and employment. Rebuilding a stronger and more resilient labour market is an important investment not only on the GDP, but also for the future and future generations. This paper considers the problem of determining the optimal sequence of control measures during the Covid-19 crisis given their impact on GDP and employment, assuming the economic system is a stochastic process. We derive the GDP (performance function) as an argument of the dynamics of the labour market during the Covid-19 crisis using the United Kingdom (UK) data to illustrate our hypothesis. The objective is to maximize the expected GDP growth during the Covid-19 crisis. We formulate this problem as a stochastic control problem, which in turn leads to a (nonlinear) quasi-variational Hamilton–Jacobi–Bellman inequality (QVHJBI).

Keywords: Optimal stochastic control, Gross Domestic Product, Hamilton-Jacobi-Bellman equation, Quasi-variational inequality

Category: **Financial Mathematics**

****

Optimal Portfolio Selection of a Constant Proportion Portfolio Insurance when Asset follows Hawkes-Jump-Diffusion

Paulina Nangolo[a], **Elias Rabson Offen**[b]
[a] University of Botswana
paulinanangolo2@gmail.com
[b] University of Botswana
offen@ub.ac.bw

Abstract

We study the firmly of risk asset in the constant proportion portfolio insurance (CPPI) trading strategy in Hawkes-jump-diffusion model where the price of the underlying asset may experience negative jump. We solve the dynamic of risk asset and cushion by using a mean version stochastic differential Equation under Geometric Brownian Motion. The main goal of portfolio insurance is to protect investor against adverse market movement. Therefore the investor choose the floor level depends on her risk preference and always try to mantain it through-out the trading period up to the maturity date, so that the Portfolio value will always lies above it. Within this framework we consider the problem of optimal Portfolio construction through the dynamics programming and its associate HJB equation of a two-dimensional to solve the the supreme of portfolio weights by considering an investors of log, power and exponential utility function. It was observed that, the higher the value of volatility and jump size, the less the expected terminal portfolio. Therefore the best payoff can be achieved with the increase in number of rebalancing, the optimal portfolio weights. Whatever types of investors we have under the utility function, the investor would rather stake his wealth in a non-jump model rather than in the discontinuous one.

Keywords: Portfolio Insurance, Optimal Portfolio, CPPI, Mean Version Stochastic differential equation, Hawkes Process

Category: **Mathematics of Finance**

References

[1] Anil K, Dietmar M and Nick C (2008). Constant proportion portfolio insurance: Statistical properties and practical Implications.*Centre for computational Finance and economic agents*.

[2] Ait-Sahalia Y, Cacho-Diaz J and Laeven (2015). *Modeling financial contagion using mutually exciting jump process. J. Financ. Eco., 117, 585-606*.

****

A model of portfolio-consumption on infinite time horizon

Siboniso Confrence Nkosi[a], **Farai Julius Mhlanga**[b] & **Mihail Zervos**[c]
[a] University of Limpopo, South Africa
siboniso.nkosi@ul.ac.za

[b] University of Limpopo, South Africa
farai.mhlanga@ul.ac.za

[c] London School of Economics and Political Science, United Kingdom
mihalis.zervos@gmail.com

Abstract

We consider the optimisation problem faced by a single agent who possesses wealth which may be consumed or invested over time. The objective is to maximise the total utility of terminal wealth over all portfolio allocation and consumption rate processes over an infinite time horizon. This portfolio optimisation problem is formulated as a singular stochastic control problem and is solved using dynamic programming approach. A verification theorem when the state process is a skew geometric Brownian motion is established. For utility functions of logarithmic type, we calculate the optimal investment and consumption policies.

Keywords: Portfolio optimisation, Skew geometric Brownian motion, Verification theorem, Infinite time horizon

Category: **Financial Mathematics**

****

Continuous-time principal agent models with long-term average payoffs

Tlou Luzarus Kubjana[a], **Farai Julius Mhlanga**[b] & **Mihail Zervos**[c]
[a] University of South Africa, South Africa
kubjatl@unisa.ac.za

[b] University of Limpopo, South Africa
farai.mhlanga@ul.ac.za

[c] London School of Economics, United Kingdom
mihalis.zervos@gmail.com

Abstract

We consider an economic project run by a sequence of managers with identical preferences and owned by a principal. We model the cashflow that the project's operation generates by either a one-dimensional arithmetic Brownian motion or an ergodic one-dimensional diffusion. We make the standard assumption that the managers are more impatient than the principal. The principal's as well as the managers' goals are to maximise their respective performance criteria.

The aim is to construct a sequence of contracts that are incentive compatible and maximise the ergodic performance criterion of the principal. To this end, we formulate the mathematical model, we characterise a class of contracts that admit a state space representation, we determine a sub-class of contracts that are incentive compatible and we reduce the problem of determining the optimal incentive compatible contract to a singular stochastic control problem that we solve.

Keywords: continous-time models, principal-agent models, incentive compatible contract, singular stochastic control.

Category: **Mathematical Finance**

References

[1] Anderson, R.W., Bustamante, M.C., Guibaud, S. and Zervos, M., 2018. Agency, firm growth, and managerial turnover. The Journal of Finance, 73(1), pp.419-464

[2] Bolton, P., and M. Dewatripont. Contract theory. MIT press, 2005.

[3] Cvitanić, J., Possamaï, D. and Touzi, N., 2017. Moral hazard in dynamic risk management. Management Science, 63(10), pp.3328-3346.

[4] Cvitanić, J., Possamaï, D. and Touzi, N., 2018. Dynamic programming approach to principal–agent problems. Finance and Stochastics, 22(1), pp.1-37.

[5] DeMarzo, P. and Sannikov, Y., 2008. Learning in dynamic incentive contracts. mimeo, Stanford University.

****

Reflected BSDE under Jacod parameters when the obstacle is right upper semi-continuous

Winfrida Felix Mwigilwa[a], **Jane Aduda**[b], **Martin B. Mbele**[c] & **Ananda Kube**[d]

[a] Pan African University Institute of Basic Sciences Technology and Innovation, Kenya

winfridamwigilwa184@gmail.com

[b] Jomo Kenyatta University of Agriculture and Technology , Kenya

jaduda@jkuat.ac.ke

[c] University of Yaounde I, Cameroon

mbelebidima@gmail.com

[d] Kenyatta University, Kenya

oakube3@googlemail.com

Abstract

In this study, there is a contribution to the theory of reflected backward stochastic differential equations (RBSDEs) when the lower barrier is considered to be right upper semi-continuous but not necessarily right-continuous. In contrast to other studies that consider Brownian motion and the Poisson process, RBSDE was formulated by using stochastic Jacod parameters to provide the important results. There is the establishment of a required a priori estimate of RBSDE first, and then the uniqueness and existence of solutions under the Lipschitz driver assumption are made. During the calculations, Mertens' decomposition of strong optional supermartingales (generalizing Doob- Meyer decomposition) is used. Furthermore, an appropriate generalization of Ito's formula due to strong optional semimartingale is combined with other techniques from optimal stopping theory. Finally, we presented an example of an application to illustrate our obtained results.

Keywords: Jacod parameters, Reflected Backward stochastic differential equations , Right upper semi-continuous, Mertens' decomposition, Random measure

References

[1] Hamadene, S., Ouknine, Y. (2016): Reflected backward sdes with general jumps. *The Journal of theory of Probability its applications 60, 263 - 280.*

[2] Baadi, B., Ouknine, Y. (2018): Reflected bsdes when the obstacle is not right-continuous in a general filtration. *arXiv preprint arXiv:1812.07990 14, 201.*

[3] Grigorova, P., Imkeller, E. O., Ouknine, Y, and Quenez, M. C., (2017): Reflected bsdes when the obstacle is not right-continuous and optimal stopping. *The Annals of Applied Probability 27, 3153-3188.*

[4] Choulli, T. and Schweizer, M. (2015): Locally phi-integrable sigma-martingale densities for general semimartingales. *Swiss Finance Institute Research Paper, 15-15.*

****

9 Fluid Dynamics

On the transformation of boundary value problems to initial value problems: The Iterative transformation method

Adewunmi Gideon Fareo

[a] University of the Witwatersrand, Johannesburg

Adewunmi.Fareo@wits.ac.za

Abstract

It is always so much desired to be able to convert a boundary value problem to an initial value problem. This is because it is easier to deal with an initial value problem than a boundary value problem. Several authors on the subject of transformation of boundary value problems to initial value problems have provided us with transformation groups that make it possible for such conversions. So far, the one-parameter scaling group and the one-parameter spiral group of transformations are the transformation groups that have been used to convert a boundary value problem to its initial value problem equivalent when it is possible to do so . Thus, there is a class of boundary value problems for which such transformation under a one-parameter group applies. When boundary value problems are not invariant under a one-parameter group, it has being shown that a modified boundary value problem can be derived, which is invariant under an extended transformation group. In this study, a concise review is presented, of how the scaling transformation group can be employed to determine a modified boundary value problem which is invariant under an extended two-parameter scaling group. The main contribution is then to show that since the non-physical scaling parameter in the extended group is unity, a simple iterative method which does not require an extended group can be deduced.

Keywords: Boundary layer theory, non-Iterative transformation method, Iterative transformation method, Scaling group.

Category: **Fluid Dynamics**

****

Bulk-surface finite element methods for non-linear semi- parabolic partial differential equations of reaction-diffusion type

Anotida Madzvamuse[a], **Lazarus Rundora**[b] & **Nkhoreng Hazel Sekgothe**[c]

[a] University of Sussex, United Kingdom
a.madzvamuse@sussex.ac.uk

[b] University of Limpopo, South Africa
lazarus.rundora@ul.ac.za

[c] University of Limpopo, South Africa
hazel.sekgothe@ul.ac.za

Abstract

We develop bulk-surface finite element methods for generalised reaction-diffusion systems and explore higher order finite elements disctretisations. The finite element algorithms will be implemented in FeNiCs.

Keywords: Bulk-surface reaction-diffusion systems, Finite element method.

Category: **Fluid Dynamics**

****

Computational Identification of Cancer Immunotherapy Targets using Combinatorial Peptide Libraries

Barbara Szomolay
Division of Infection and Immunity, School of Medicine, Cardiff University, United Kingdom
szomolayb@cardiff.ac.uk

Abstract

The interaction between T-cell receptors (TCRs) and peptides is highly degenerate: a single TCR may recognize about one million different peptides in the context of a single MHCI molecule. On the other hand, TCR recognition is fundamentally peptide- and/or MHC-specific: the functional sensitivity, which can be viewed as experimental realisation of the TCR triggering rate, is large enough only for minute fraction of all possible ligands. TCR triggering rate and degeneracy are mathematical concepts that are fundamental for an approach that uses length-matched combinatorial peptide library (CPL) scan data to search protein databases and to rank peptides in order of likelihood of recognition. This CPL-based database screening can, to a large extent, accurately identify self-peptides that triggered the CD8 T-cell. The computational time required for peptide searching can be significantly reduced by using graphics processing units (GPUs). Adoption of GPU-accelerated prediction of T-cell agonists has the capacity to revolutionise our understanding of cancer immunity by identifying potential targets for tumor-specific T-cells.

Keywords: functional sensitivity, peptide libraries, GPU

Category: **Computational Biology**

References

[1] 8. Whalley T, Dolton G, Brown PE, Wall A, Wooldridge L, van den Berg HA, Fuller A, Hopkins JR, Crowther MD, Attaf M, Knight RR, Cole DK, Peakman M, Sewell AK, Szomolay B, GPU-accelerated discovery of pathogen-derived molecular mimics of a T-cell insulin epitope (2020) Frontiers in Immunol 11:296

****

Closed-form solutions and conserved quantities for a three-dimensional soliton equation

Chaudry Masood Khalique[a]
[a]International Institute for Symmetry Analysis and Mathematical Modelling,
Department of Mathematical Sciences, North-West University,
Mafikeng Campus, Private Bag X 2046, Mmabatho 2735,
Republic of South Africa
Masood.Khalique@nwu.ac.za

Abstract

In this talk, we study and analyze a three-dimensional soliton equation, which has applications in fluid mechanics and other nonlinear sciences such as plasma physics, atomic physics, biophysics, nonlinear optics, classical and quantum fields theories. We perform symmetry reductions of the equation using Lie group theory and obtain analytic solutions through this technique. Direct integration of the resulting ordinary differential equation is done which gives new analytic travelling wave solutions that consist of rational function, elliptic functions, elementary trigonometric and hyperbolic functions solutions of the equation. We also contemplate conserved quantities for the equation under study using the multiplier approach.

Keywords: Soliton equation, Lie group analysis, symmetries, conservation laws, multiplier method

Category: **Fluid Dynamics**

References

[1] Liu J., Zhang Y. (2018): Construction of lump soliton and mixed lump stripe solutions of (3+1)-dimensional soliton equation. *Results in Physics*.

[2] Drazin P.G., Johnson R.S. (1989): Solitons: An Introduction. *Cambridge University Press, New York*.

[3] Olver P.J. (1993): Applications of Lie Groups to Differential Equations. *second ed., Springer-Verlag, Berlin*.

****

Exact solutions and conservation laws of Boussinesq equation and extended Kadomtsev-Petviashili equation in fluids

Christina. Majola, **Letlhogonolo .D. Moleleki** & **Masood Khalique**
International Institute for Symmetry Analysis and Mathematical Modelling, Department of Mathematical Sciences, North-West University, Mafikeng Campus, Private Bag X 2046, Mmabatho 2735,
Republic of South Africa
Email: 25278045@nwu.ac.za, Letlhogonolo.Moleleki@nwu.ac.za, Masood.Khalique@nwu.ac.za

Abstract

In this study a nonlinear evolution partial differential equations, namely, (1+1)-Dimensional Bossinesq equation and (3+1)-Dimensional Bossinesq equation. Lie symmetry method together with Simplest equation method and Kudryoshov's method is used to find the exact solutions of the (1+1)-dimensional Boussinesq and (3+1)-Dimensional Bossinesq equation. Furthermore,

multiplier method will be used to construct the conservation laws of the the above mentioned equations.

Keywords: (1+1)-dimensional Boussinesq equation, (3+1)-dimensional Bossinesq equation, Lie point symmetries,Simplest equation method,Kudryoshov's method, conservation laws

Category: **other**

References

[1] A. M. Wazwaz, L. Kaur, New integrable Boussinesq equations of distinct dimensions with diverse variety of soliton solutions,Nonlinear Dyn, 97,83-94 (2019).

[2] C. Gu, H. Hu, Z. Zhou, Darboux Transformation in Soliton Theory and Its Geometric Applications, Springer, The Netherlands, 2005.

[3] M.J. Ablowitz, P.A. Clarkson, Solitons, Nonlinear Evolution Equations and Inverse Scattering, Cambridge University Press, Cambridge, UK, 1991.

[4] R. Hirota, The Direct Method in Soliton Theory, Cambridge University Press, Cambridge, 2004.

****

Error estimates for pressure-driven Hele-Shaw flow

John Fabricius[a], **Salvador Manjate**[a,b] & **Peter Wall**[a]
[a] Department of Engineering Sciences and Mathematics
Luleå University of Technology
SE-971 87 Luleå, Sweden
john.fabricius@ltu.se & peter.wall@ltu.se
[b] Department of Mathematics and Informatics
Eduardo Mondlane University
Box 257 Maputo, Mozambique
salvador.manjate@ltu.se

Abstract

We consider Stokes flow past cylindrical obstacles in a generalized Hele-Shaw cell, i.e. a thin three-dimensional domain confined between two surfaces. The flow is assumed to be driven by an external pressure gradient, which is modeled as a normal stress condition on the lateral boundary of the cell. On the remaining part of the boundary we assume that the velocity is zero. We derive a divergence-free (volume preserving) approximation of the flow by studying its asymptotic behavior as the thickness of the domain tends to zero. The approximation is verified by error estimates for both the velocity and pressure in H^1- and L^2-norms, respectively.

Keywords: Hele-Shaw flow, asymptotic expansions, pressure boundary condition, thin film flow, error estimates.

Category: **Fluid Dynamics**

References

[1] G. Bayada, M. Chambat. (1986): The transition between the Stokes equations and the Reynolds equation: A mathematical proof, *Appl. Math. Optim.,* 14(1):73–93.

[2] J. Fabricius, S. Manjate, P. Wall.(2021): On pressure-driven Hele-Shaw flow of power-law fluids, *Applicable Analysis*, DOI: 10.1080/00036811.2021.1880570.

[3] J. Fabricius, E. Miroshnikova, A. Tsandzana, P. Wall. (2020): Pressure-driven flow in thin domains, *Asymptot. Anal.*, 116(1):1–26.

[4] S. Marušić, E. Marušić-Paloka.(2000): Two-scale convergence for thin domains and its applications to some lower-dimensional models in fluid mechanics, *Asymptot. Anal.* 23(1):23–57.

[5] J. Wilkening. (2009): Practical error estimates for Reynolds' approximation and its higher order corrections, *SIAM J. Math. Anal.*, 41(2):588-630.

****

Solution of Partial Differential Equations using Adomian Decomposition Method

Julius Maremwa Shichikha[a] & **Albert Bii**[b]

[a]University of Eldoret, Department of Mathematics and Computer Science,
P. O. Box 1125-30100, Eldoret-Kenya
jmaremwa@yahoo.com

[b] University of Eldoret, Department of Mathematics and Computer Science,
P. O. Box 1125-30100,Eldoret-Kenya
albert.bii@uoeld.ac.ke

Abstract

Partial differential equations occur frequently in Science and Engineering. Analytical solutions are generally hard to obtain. In this paper, Adomian Decomposition Method [1] is applied to solve one and two dimensional heat [2] equations. The method decomposes the solution into convergent series. The solution if available in closed form, the series will converge to it. The solutions so obtained are represented graphically.

Keywords: Adomian Decomposition Method, Convergent series.

Category: **Fluid Dynamics**

References

[1] Adomian, G. (1994) : *Solving Frontier Problems of Physics : The Decomposition Method*. Kluwer Academic Press.

[2] Biazar, J . and Amirtaimoori, A. R. (2005) : An Analytical Approximation to the Solution of heat equation by Adomian Decomposition Method and Restrictions of the Method: *Applied Mathematics and Computation* 171 , 738-745.

****

Lie algebra, group-invariant and analytic solutions of a generalized (3+1)-dimensional nonlinear potential Yu-Toda-Sasa-Fukuyama equation in Mathematical Physics

Oke Davies Adeyemo[a]
[a]International Institute for Symmetry Analysis and Mathematical Modelling,
Department of Mathematical Sciences, North-West University,
Mafikeng Campus, Private Bag X 2046, Mmabatho 2735,
Republic of South Africa
adeyemodaviz@gmail.com

Abstract

In this talk, we present the analytical investigation of a generalized (3+1)-dimensional nonlinear potential Yu-Toda-Sasa-Fukuyama equation which has applications in the fields of engineering and physics. The theory of Lie group is applied to the nonlinear partial differential equation to basically reduce the equation to an integrable form which consequently allows for direct integration of the result. The Lie algebra of the model makes it possible to achieve various nontrivial solutions. Besides, more general solutions are found through a well-known standard technique. Conclusively, we construct conserved quantities of the underlying equation.

Keywords: generalized (3+1)-dimensional nonlinear potential Yu-Toda-Sasa-Fukuyama equation, group-invariant solutions, Lie group analysis, conservation laws

Category: **Fluid Dynamics**

References

[1] Gu Y., Qi J. (2017): Symmetry reduction and exact solutions of two higher-dimensional nonlinear evolution equations. *Math. Inequal. Appl.*

[2] Hu Y., Chen H., Dai Z. (2014): New kink multi-soliton solutions for the (3+1)-dimensional potential-Yu-Toda-Sasa-Fukuyama equation. *Appl. Math. Comput.*.

[3] Olver P.J. (1993): Applications of Lie Groups to Differential Equations. *second ed., Springer-Verlag, Berlin*.

****

Highly Accurate Compact Finite Difference Schemes for Two-Point Boundary Value Problems with Robin Boundary Conditions

Phumlani Dlamini
University of Johannesburg, South Africa
phgdlamini@gmail.com

Abstract

he finite difference method is among the oldest and the most used approaches to approximate solutions of differential equations. Its major disadvantage is that it usually needs many grid points to give acceptable approximations, often leading to large memory requirements. Of late, compact finite difference methods have become popular among researchers solving differential equations. That is because they provide high-order accuracy for far fewer grid points than the common classical finite differences. In this talk, we present derivations of high-order compact finite difference schemes. We also discuss compact finite difference schemes for the functions with Dirichlet, Neumann, and Robin boundary conditions. Lastly, we show how the compact finite difference schemes are used to solve various ordinary and partial differential equations subjected to different types of boundary conditions.

Keywords: boundary value problems, compact finite differences, quasilinearization, Robin boundary conditions

****

Numerical study of convective flow of a combustible third grade fluid through an inclined channel with variable porous permeability and viscosity

Samuel Olumide Adesanya[a], **Lazarus Rundora**[b] & **Kgomotshwana Frans Thosago**[c]
[a] Redeemer's University, Nigeria
adesanyas@run.edu.ng
[b] University of Limpopo, South Africa
lazarus.rundora@ul.ac.za
[c] University of Limpopo, South Africa
frans.thosago@ul.ac.za

Abstract

A mathematical examination for convective flow of combustible third-grade fluid through an inclined vertical channel exposed to Newtonian cooling is conducted in the present study. The

fluid viscosity and permeability of the porous medium are assumed to follow exponential variation with temperature. The mixed convective flow is attributed to pressure difference at the inflow and out ends of the channel in the direction of flow together with thermal effects on the density of the viscous combustible liquid. The channel walls are assumed to have convective air exchange with the ambient. Based on the nonlinear physical properties of the combustible liquid, solutions of the dimensionless version of the governing equations are solved numerically by using the spectral Chebyshev polynomial as the admissible trial solution for the weighted residual method. The computations were validated with shooting Runge-Kutta approach. Fluid velocity and temperature profiles and the implications on heat irreversibility are computed with thermal critical values. Graphical and tabular illustrations are provided with adequate explanations. The out-coming results are useful in achieving efficient energy utilization processes and equipment designs.

Keywords: reactive fluids; entropy generation; third-grade fluid; porous medium; heat flux; fluid slippage

Category: **Fluid Dynamics**

References

[1] S.O. Salawu, A.R. Hassan, A. Abolarinwa, N.K. Oladejo, Thermal stability and entropy generation of unsteady reactive hydromagnetic Powell-Eyring fluid with variable electrical and thermal conductivities, *Alexandria Engineering Journal (2019) 58, 519–529*.

[2] Samuel O. Adesanya, Semiu O. Kareem, John A. Falade, Samson A. Arekete(2015), Entropy generation analysis for a reactive couple stress fluid flow through a channel saturated with porous material, *Energy, 93, 1239 – 1245*

[3] Mondal, H.; Mishra, S.; Kundu, P.K.; Sibanda, P.Entropy generation of variable viscosity and thermal radiation on magneto nanofluid flow with dusty fluid. *Journal of Applied and ComputationalMechanics2020,6,171–182*.

****

A new overlapping grid approach for the solution of first order IVPs using hybrid block methods

Sandile Motsa[1]
[1]University of Eswatini, Department of Mathematics, P/Bag 4, Kwaluseni, Eswatini
E-mail address: sandilemotsa@gmail.com

ABSTRACT

This work presents a new modification of the hybrid block methods (HBM) for solving Initial Value Problems (IVPs). The new approach, called the overlapping hybrid block method (OE-HBM) is developed by modifying each integrating block of the HBM to include the penultimate intra-step point of the preceding block. In this study, we present preliminary results obtained

from applying the OEHBM, with equally-spaced grid points, on first order IVPs. To highlight the improvement of the proposed method, the results have been compared with the equivalent HBM based on the standard non-overlapping grid. Comparisons, based on truncation errors, stability analysis and numerical errors on specific examples are presented. The proposed OE-HBM is found to be more accurate than existing HBM variants of the same order.

Keywords: *hybrid block methods; overlapping grid; Collocation;*

Category: **Numerical Methods**

****

Southern Africa Mathematical Sciences Association Annual Conference - SAMSA2022

November 22 - 25, 2022, Maputo, Mozambique

A numerical analysis on the unsteady flow of a thermomagnetic reactive Maxwell nanofluid over a stretching/shrinking sheet with Ohmic dissipation and Brownian motion

Stanford Shateyi[a],& **Hillary Muzara**[b]

[a] University of Venda

stanford.shateyi@univen.ac.za

[b] University of Zimbabwe

muzarahillary@gmail.com

Abstract

The major objective of this current investigation is to examine the unsteady flow of a thermomagnetic reactive Maxwell nanofluid flow over a stretching/shrinking sheet with Ohmic dissipation and Brownian motion. Suitable similarity transformations were used to reduce the governing nonlinear partial differential equations of momentum, energy and species conservation into a set of coupled ordinary differential equations. The reduced similarity ordinary differential equations were then solved numerically using the Spectral Quasi-Linearization Method. The influence of some pertinent physical parameters on the velocity, temperature and concentration distributions was studied and analysed graphically. Further investigations were made on the impact of the Eckert number, Prandtl number, Schmidt number, thermal radiation parameter, Brownian motion parameter, thermophoresis parameter and chemical reaction parameter on the skin friction coefficient, surface heat and mass transfer rates. The results were displayed in a tabular form.

The flow which is in the upper region $y > 0$ and has a uniform magnetic field of strength $B = \frac{B_0}{\sqrt{1-\alpha t}}$ subjected to it, with an initial magnetic field strength denoted by B_0. The magnetic field is applied perpendicular to the sheet. The unsteady fluid, heat and mass transfer flows are assumed to start at $t = 0$. The sheet emerges at point $(x, y) = (0, 0)$ moving with a non-uniform velocity $u_w(x, t) = \frac{ax}{1-\alpha t}$, where a and α are positive constants with inverse time dimension $(time)^{-1}$, and a is an initial stretching rate. The velocity of mass transfer is $v_w(x, t) = \frac{v_0}{\sqrt{1-\alpha t}}$, where v_0 is the constant mass flux velocity. Also, T_w and C_w are the surface temperature and concentration, respectively. Then T_∞ and C_∞ are their respective ambient values. In this problem, the induced magnetic field is considered negligible due to the magnetic Reynolds number in the flow which is assumed to be very small. Also, assumed to be negligible is the external electric field and the electric field which result from the polarization of charges. Therefore, the governing equations of such a model flow, under the usual boundary layer approximations in the usual notations are, Madhu *et al.* [2]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right) = \nu\frac{\partial^2 u}{\partial y^2} + g\beta_f(T - T_\infty) + g\beta_f^*(C - C_\infty) - \frac{\sigma B^2}{\rho_f}u, \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_f}{(\rho c)_f}\frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho c)_f}\frac{\partial q_r}{\partial y} + \frac{\mu_f}{(\rho c)_f}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B^2}{(\rho c)_f}u^2 + \tau\left\{D_B\frac{\partial T}{\partial y}\frac{\partial C}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\}, \tag{3}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} - k_r(C - C_\infty), \tag{4}$$

The appropriate boundary conditions for the model under consideration are given by:

$$u = u_w(x,t),\ v = v_w(x,t),\ T = T_w(x,t),\ C = C_w(x,t), D_B\frac{\partial C}{\partial y} + \frac{D_T}{T_\infty}\frac{\partial T}{\partial y} = 0 \quad at\ y = 0, \tag{5}$$
$$u \to 0,\ \ C \to C_\infty,\ \ T \to T_\infty,\ \ \text{as } y \to \infty.$$

Where $T_w(x,t) = T_\infty + aT_0(1-\alpha t)^{-\frac{3}{2}}/(2\nu_f),\ \ C_w(x,t) = C_\infty + aC_0(1-\alpha t)^{-\frac{3}{2}}/(2\nu_f)$, with T_0 and C_0 being reference, temperature and concentration, respectively.

Keywords: Maxwell nanofluid; Stretching sheet; Ohmic dissipation; Brownian motion; Spectral quasi-linearization; Gauss-Lobatto points .

Category: **Fluid Dynamics**

References

[1] Kierzenka, J. & Shampine, L. F. A BVP solver based on residual control and the MATLAB PSE. ACM Trans. Math. Soft. 27(3), 299316 (2001).

[2] Madhu, Macha; Kishan, Naikoti & Chamkha, Ali J. (2017). Unsteady flow of a Maxwell nanofluid over a stretching surface in the presence of magnetohydrodynamic and thermal radiation effects. Propulsion and Power Research, 6(1), 3140. doi:10.1016/j.jppr.2017.01.002.

[3] El-Aziz, M. A., & Yahya, A. S. (2017). Heat and Mass Transfer of Unsteady Hydromagnetic Free Convection Flow Through Porous Medium Past a Vertical Plate with Uniform Surface Heat Flux. Journal of Theoretical and Applied Mechanics, 47(3), 2558.

[4] Daniel, Yahaya Shagaiya; Aziz, Zainal Abdul; Ismail, Zuhaila & Salah, Faisal (2017). Thermal radiation on unsteady electrical MHD flow of nanofluid over stretching sheet with chemical reaction. Journal of King Saud University - Science, (), S1018364717308583. doi:10.1016/j.jksus.2017.10.002

[5] Bellman, R. & Kalaba, R. (1965) Quasilinearization and Nonlinear Boundary-Value Problems. American Elsevier Publishing Company, New York.

[6] Trefethen, Lloyd N. Spectral methods in MATLAB. Society for Industrial and Applied Mathematics, 2000.

[7] E. M. A. Elbashbeshy & M. A. A. Bazid (2004). Heat transfer over an unsteady stretching surface. , 41(1), 14. doi:10.1007/s00231-004-0520-x.

Effect of variable viscosity on MHD micropolar hybrid nanofluid flow over a rotating cone

T. Khumalo[a] & **S. Ayano**[b]
Department of Mathematics, University of Eswatini, Eswatini
[a] thokozanik74@gmail.com
[b] mekk aya@yahoo.com

Abstract

The present paper examine a temperature-dependent viscosity and thermal conductivity of micopolar Ag(silver)-MgO(Magnesium oxide) hybrid nanofluid fluid over a rotating vertical cone with the influence of transverse magnetic field and thermal radiation. Hybrid nanofluid is the extension towards nanofluid with two or more dilute suspension of composite nanoparticles (accessible and relatively cheap) in a base fluid. Physical flow problem has been modled with coupled partial differential equations. We apply similarity transformations to the non-dimensionalized equations and the resulting non linear differential equations are solved numerically. The flow behavior for fluid was scrutinized under the impact of diverse physical constraints are illustrated graphically.

Keywords: Human papillomavirus virus, Vaccination rate, Dual-rate effect, Numerical simulations.

Category: **Mathematical Biology**

References

[1] Saldana, F., Korobeinikov, A., and Barradas, I. (2019): Optimal control against the human papillomavirus: protection versus eradication of the infection. *Abstr. Appl. Anal., 1-13*.

[2] World Health Organization: Global strategy to accelerate the elimination of cervical cancer as a public health problem. https://www.who. int/publications/i/item/9789240014107. Accessed 05 March 2022.

[3] Genital HPV infection: CDC fact sheet. Centers for Disease Control and Prevention. http://www.cdc.gov/std/HPV/STDFact-HPV.htm. Accessed 11 June 2022.

[4] Oduro, B. and Magagula, V. M. (2021): COVID-19 intervention models: An initial aggressive treatment strategy for controlling the infection. *Infectious Disease Modelling, 6, 351-361*.

****

10 MATHEMATICAL BIOLOGY

An Agent-Based Model for Studying the Spread of COVID-19 in Population Mobility Patterns Scenarios

Alfredo Zacarias Muxlhanga & **Sansão Agostinho Pedro**
Department of Mathematics and Informatics, Eduardo Mondlane University, 257, Maputo, Mozambique
alfredo.muxlhanga@uem.ac.mz, sansao.pedro@uem.ac.mz

Daisuke Takahashi
A.I.Systems Research Co.,Ltd 4 Yoshidaushinomiya-cho, Sakyo-ku, Kyoto 606-8302, Japan
dtakahashi42@gmail.com

Abstract

During an outbreak of an infectious disease, mathematical models and computer simulations are essential tools to characterize the epidemic dynamics and assist in the formulation of public health policies [1]. The movement of each individual at any time is crucial for the transmission or contamination of an infectious disease due to different contacts in different places. In this work, we develop an agent-based model, which relates the movement patterns of people during different periods of time of each day of the week (morning, afternoon and night), in different places (home, school, work place, worship place, market and others), with the levels of compliance with measures to prevent the COVID-19 pandemic (social distancing and opening and closing paces). For the study of spatial dynamics, a survey was carried out in the city of Maputo, where it was possible to estimate a matrix of the average numbers of contacts between individuals according to the places and periods of each day of the week. For disease dynamics, the SEAIR[1] model was privileged and it was possible to calibrate the model with the proportion of diagnosed symptomatics (I), where it was observed that the trend of the graph of the real data is close to the model developed in this work. Furthermore, it was concluded that closing schools and other places is effective when levels of social distancing compliance in places that remain open are around 75% minimum. On the other hand, the closing of worship places does not have much influence in flattening the epidemic curve. This case is explained by the fact that there is a lower frequency of visits to these places, associated with a shorter length of stay. The average contact numbers are relatively equal in some locations, however, the closing of each location has a different impact on the flattening of the curve due to differences in movement patterns in large proportions.

Keywords: agent based models, COVID-19, mobility, contacts, opening and closing rhythms, social distancing.

Category: **Mathematical Biology**

[1] Susceptible-Exposed-Asymptomatic-Symptomatic-Recovered

References

[1] de Souza, S. L., Batista, A. M., Caldas, I. L., Iarosz, K. C., & Szezech Jr, J. D. (2021). *Dynamics of epidemics: Impact of easing restrictions and control of infection spread.* Chaos, Solitons & Fractals, 142, 110431.

****

On Traveling Wave Solution in Spatial Co-Infection System. The Case of Maize Lethal Necrosis

Americo Matusse[a,b], **Michael Chapwanya**[a] & **Yves Dumont**[b,c,d]

[a] Departamento de Matemática e Informática, Universidade Eduardo Mondlane, Moçambique
[b] Department of Mathematics and Applied Mathematics, University of Pretoria, 0002, South Africa

americo.matusse@uem.ac.mz

[b] Department of Mathematics and Applied Mathematics, University of Pretoria, 0002, South Africa
m.chapwanya@up.ac.za

[b] Department of Mathematics and Applied Mathematics, University of Pretoria, 0002, South Africa
[c] CIRAD,UMR AMAP, 97410 St Pierre, Réunion Island, France
[d] AMAP, University of Montpellier, CIRAD, CNRS, INRA, IRD, Montpellier, France
yves.dumont@cirad.fr

Abstract

Host infection by multiple pathogen genotypes is a crucial constrain in disease ecology and evolution. For instance, *Maize Lethal Necrosis Disease* (MLND) in Southern Africa results from synergistic interaction between the *Maize Chlorotic Mottle Virus* (MCMV) and potyviridae such as *Sugarcane Mosaic Virus* (SCMV). MCMV are transmitted by several vectors including, beetles, flower thrips and maize thrips (*Frankliniella Williams*) in semi-persistent manner while SCMV are transmitted by aphids including *Rhopalosiphum maidis, Rhopalosiphum padi, Myzus persicae, Schizaphis graminum* in nonpersistent manner.

In this talk, we propose a partially degenerate co-infection vector borne-disease model in unbounded domain. We present some theoretical results on the existence of traveling wave solution when one of the virus is circulating. When both virus are circulating, we show the existence of traveling wave solutions through numerical simulations, for different invasion scenarios depending on the threshold parameters and initial conditions. We discuss the consequences of these traveling waves in terms of crop protection.

Keywords: Crop disease, co-infection, synergistic interaction, partially degenerate reaction-diffusion system, traveling wave, numerical simulation.

Category: **Mathematical Biology**

****

Dynamics and Optimal Control Analysis of COVID-19 Transmission Epidemic Model

Amosse Chicomo & **Sansão Pedro**
Faculty of Sciences of Eduardo Mondlane University
amosse.chicomo@gmail.com

Faculty of Sciences of Eduardo Mondlane University
sansao.pedro@uem.ac.mz

Abstract

Since the end of 2019, the world has been experiencing a health system threatened by the COVID-19 disease. To understand the dynamics of the disease and create strategies for its containment, many studies have been carried out since the beginning of its existence. This study aims to analyze the epidemiological behavior of COVID-19 through the mathematical model and analyze the effect of applying optimal control. To carry out this study, the literature was used, which culminated in the assembly of the SEIAHR model studied here. In this case, the basic reproduction number (R_0) was calculated to qualitatively analyze the stability of the equilibrium points, where it was shown that if $R_0 < 1$ the disease-free equilibrium point is asymptotically globally stable and the endemic equilibrium point is asymptotically globally stable if $R_0 > 1$. Regarding the implementation of optimal control, the use of vaccination strategy applied to the control proved to be ideal for reducing the spread of COVID-19. Therefore, the control profile showed that individuals should be vaccinated throughout the duration of disease.

Keywords: Epidemic Modeling, SEIAHR, Stability anaysis, optimal control, COVID-19.

Category: **Mathematical Biology**

****

An Age-Structured Model for Transmission Dynamics of Malaria with Infected Immigrants and Asymptomatic Carriers

Asha Kalula[a,b], **Eunice Mureithi**[a], **Theresia Marijani**[a] & **Isambi Mbalawata**[c]
[a] Department of Mathematics, University of Dar es Salaam, P. O. Box 35062, Tanzania.
ewambui02@gmail.com, tmarijani@gmail.com

[b] Department of Mathematics, Physics and Information, Mkwawa University College of Education, P.O. Box 2513, Tanzania.

kalulaasha@gmail.com

[c] African Institute for Mathematical Sciences (AIMS), Cellule Kamatamu, Rue KG590 ST No 1 Kigali, Rwanda.

imbalawata@nexteinstein.org

Abstract

An age-structured (children and adults) model for the transmission dynamics of malaria with asymptomatic carriers and infected immigrants has been analyzed. We first analyze a model without infected immigrants. It shows that the disease-free equilibrium exists and is stable when $R_0 < 1$ and unstable for $R_0 > 1$. Also, we compute the sensitivity indices of the basic reproduction number. The basic reproduction number is most sensitive to the mosquito biting rate. Besides, the sensitivity of the basic reproduction number shows that the children's class parameters are more sensitive than those of adults. In the presence of infected immigrants, the model does not admit a disease-free equilibrium. The sensitivity of endemic equilibrium shows that the asymptomatic carrier parameters are more critical than that of infected immigrants. Also, the inflow of infectious immigrants is sensitive than that of infected immigrants. The results obtained indicate that

Keywords: Age-structure, malaria, immigrants, asymptomatic carrier, non-linear ODE model

References

[1] Addawe, J. M. and Lope, J. E. C. (2012a): Analysis of age-structured malaria transmission model. Philippine Sci. Lett. 5, pp. 169–186.

[2] Addawe, J. M. and Lope, J. E. C. (2012b): Sensitivity analysis of the age-structured malaria transmission Model. In AIP Conference Proceedings 1482: 47-53, American Institute of Physics.

[3] Addawe, J. and Pajimola, A. K (2016): Dynamics of climate-based malaria transmission model with the age-structured human population. AIP Conference Proceedings 1782(1): 040002, AIP Publishing LLC.

Coagulation of particles based on Smoluchowski's equation with external sources

Dmitri Alexandrov, **Eugenya Makoveeva**, **Margarita Nikishina** & **Irina Alexandrova**
Laboratory of Stochastic Transport of Nanoparticles in Living Systems, Laboratory of Multi-Scale Mathematical Modeling, Department of Theoretical and Mathematical Physics, Ural Federal University, Lenin ave., 51, Ekaterinburg, 620000, Russian Federation
dmitri.alexandrov@urfu.ru

Abstract

The coagulation equation, proposed by Smoluchowski, has been widely used to describe aggregation phenomena in many fields of science since its inception. It considers a physical system of many nanoparticles, and each particle is characterized by a change of some non-negative scalar quantity (e.g. volume). Assuming such a system to be spatially homogeneous and unbounded, considering only pairwise interactions and a balance relation of interacting particles, the Smoluchowski equation can be used to describe the evolution of a system of many

nanoparticles. In this study, we construct an exact solution to this integro-differential equation containing an exponentially-decaying source term. This solution, in particular, describes the steady-state structural density of endosomes per cell carrying the nanoparticles. Also, we show that our exact solution well describes recent experimental data on gold nanoparticle distribution inside endosomes. In addition, we derive an analytical solution to the non-stationary coagulation equation stitching the steady-state and initial distributions of structural density. Choosing a particular form of stitching functions, we demonstrate that the total number of nanoparticles inside a cell (calculated based on this solution) is in good agreement with recent experiments in living cells. Thus, analytical solutions obtained represent a general theoretical basis to describe the dynamics of cargo distributions in the endosomal network.

This work was supported by the Russian Science Foundation (Grant number 18-19-00008).

Keywords: Coagulation, Nanoparticles, Structural density, Analytical solutions, Endosomal network

Category: **Mathematical Biology**

****

On the impact of hierachy of resort treatment-seeking behavior on malaria infection

Faraimunashe Chirove[a]
[a] University of Johannesburg
fchirove@uj.ac.za

Abstract

Malaria is still a huge public health problem and many lives would be saved if access to good quality, affordable and efficacious medicines were readily available. Antimalarial drugs form part of a wider set of products where quality is not directly observable at the time of purchase, and only partially observable when used. Fake antimalarial drugs widespread use due to hierachy of resort treatment seeking behaviour has become a threat towards malaria treatment efforts with a longrun possibility of adverse effects and development of drug resistance. The study focuses on quantifying the veracity of using fake malaria drugs driven by hierachy of resort treatment seeking bahavior on malaria disease using a deterministic and stochastic model. Deterministic and stochastic thresholds were computed that determine the conditions for disease progression or extinction. Our results suggest that the chances of disease extinction are reduced and time to extinction is prolonged when access to fake drugs is prioritised. Efforts should be dirrected towards intercepting the channels suppying fake drugs in communities.

Keywords: Malaria, Fake drugs, Hierachy of resort, Probability of extinction

Category: **Mathematical Biology**

****

Modelling Botswana's HIV/AIDS response and treatment policy changes: Insights from mathematical models

Farai Nyabadza[a] & **Tefa Kaisara**[b]
[a] University of Johannesburg
fnyabadza@uj.ac.za

[b] Botswana International University of Science and Technology
kaisarat@biust.ac.bw

Abstract

The management of HIV/AIDS has evolved ever since advent of the disease in the past three decades. Many countries have had to revise their policies as new information on the virus, and its transmission dynamics emerged. In this paper, we track the changes in Botswana's HIV/AIDS response and treatment policies using a piece-wise system of differential equations. The policy changes are easily tracked in three epochs. Models for each era are formulated from a [**?**] that can be linked to all the epochs. The grand model's steady states are determined and analysed in terms of the model reproduction number, R_0. The model exhibits a backward bifurcation, where a stable disease-free equilibrium coexists with a stable endemic equilibrium when $R_0 < 1$. The stability of the models for the other epochs can be derived from that of the grand model by setting some parameters to zero. The models are fitted to HIV/AIDS prevalence data from Botswana for the past three decades. The changes in the populations in each compartment are tracked as the response to the disease and treatment policy changed over time. Finally, projections are made to determine the possible trajectory of HIV/AIDS in Botswana. The implications of the policy changes are easily seen, and a discussion on how these changes impacted the epidemic are articulated. The results presented have crucial impact on how policy changes affected and continue to influence the trajectory of the HIV/AIDS epidemic in Botswana.

Keywords: HIV/AIDS; Grand model; Reproduction number; Treatment policy; Fitting; Simulations

Category: **Mathematical Biology**

References

[1] Nyabadza F, Mukandavire Z, Hove-Musekwa SD. Modelling the HIV/AIDS epidemic trends in South Africa: Insights from a simple mathematical model. *Nonlinear Analysis: Real World Applications*. 12(4):2091-104, (2011).

[2] Rahman SM, Vaidya NK, Zou X. Impact of early treatment programs on HIV epidemics: An immunity-based mathematical model, *Mathematical Biosciences*, 280:38-49,(2016).

[3] Manda EC, Chirove F. Modelling coupled within host and population dynamics of R_5 and X_4 HIV infection, *Journal of Mathematical Biology* 76:1123-1158,(2018).

****

Exploring the impact of human behaviour on the spread and control of infectious diseases: how mathematical modelling can help

K A Jane White[a,b], **Farai Chirove**[b] & **Suzanne Lenhart**[c]
[a] Department of Mathematical Sciences, University of Bath, UK.
maskajw@bath.ac.uk

[b] Department of Applied Mathematics, University of Johannesburg, South Africa
fchirove@uj.ac.za

[c] Department of Mathematics, University of Tennessee, Knoxville, USA
slenhart@tennessee.edu

Abstract

The COVID-19 pandemic highlighted to everyone the important role that individual behaviour plays in the spread of an infectious disease through a population, and also the contribution that mathematical models can make to understanding infection dynamics and control. In this talk we will outline a number of qualitatively different approaches to incorporate individual behaviour into a mathematical model of infectious disease. In each case, we will explore the assumptions that are being used and will identify the types of data that are required to validate the approach. We will provide examples of behaviour-mediated infectious disease modelling that we have undertaken to explore particular problems in different African countries (see, for example [1], [2]). We will highlight the data-driven constraints which we needed to employ for model validation, and will demonstrate how our different modelling approaches form part of a mathematical toolbox for public health infectious disease control teams. We will conclude by reflecting on our experiences of modelling infectious disease dynamics in African contexts and using the reflections to identify priorities for future modelling activity.

Keywords: Infectious disease, individual behaviour, control, mathematical model.

References

[1] Levy, B., Correia, H.E., Chirove, F., Ronoh, M., Abebe, A., Kgosimore, M., Chimbola, O., Machingauta, M.H., Lenhart, S., White, K.A.J. (2021): Modeling the Effect of HIV/AIDS Stigma on HIV Infection Dynamics in Kenya.*Bull Math Biol* 83, 55.

[2] Edholm, C.J., Levy, B. Spence, L., Agusto, F.B., Chirove, F., Chukwu, W., Goldsman, D., Kgosimore, M., Maposa, I., White, K.A.J., Lenhart, S. (2022): A vaccination model for COVID-19 in Gauteng, South Africa. *Infectious Disease Modelling* 7(3): 333-345

****

On COVID-19 transmission dynamics, number of waves and impact of vaccination: Mathematical modeling, analysis and numerical simulations

J. Malinzi, V.O. Juma, C.E. Madubueze, J. Mwaonanji, G.N. Nkem, E. Mwakilama, T. Mupedza, V. Nandwa, E.A. Bakare, I.L. Moyo-Zulu, E. Campillo-Funollet, T. Mazikana, S. Osman, A. Madzamuse and F. Nyabadza

Abstract

This study aims to evaluate the epidemic trend of the COVID- 19 epidemic amid increasing vaccination coverage. The potential epidemic trends, with current vaccination rates, are determined and plausible vaccination rates and efficacies are suggested in order to obtain better results. In doing so, we seek to address the following questions:

1. What are the current COVID-19 transmission dynamics?
2. What fraction of a country's population should be vaccinated, and with what efficacy, in order to obtain optimal results?
3. How long could a wave be, going by the current vaccination rates?

A compartmental mathematical model is developed, analysed, calibrated and numerically simulated to investigate the dynamics, impact of vaccination and control interventions of the Coronavirus disease and determine the effect of vaccine efficacy. A thorough qualitative analysis of the model is carried out to infer the possible long term possibilities of the populations. The next generation matrix is used to obtain the basic reproduction number R_0. The model equilibria and their stability, which depends on R_0, are presented. The model was calibrated using South African reported data for COVID-19. Plausible parameter values have been determined and several numerical simulations have been carried out. Both local and global sensitivity analyses with respect to the basic reproduction number have been carried out to determine the most influential parameters that most dictate the epidemic.

This project is being undertaken by several collaborators from the GCRF-EPSRC UK-APASI program [1]. Results that have been obtained thus far will be presented. We hope that the results could help to guide policy makers and governments to make plausible decisions especially to do with designing vaccination programs and implementation of other mitigation strategies such that the economies are not negatively substantially affected.

Keywords:COVID-19, Qualitative analysis, Number of waves

Category: **Mathematical Biology**

References

[1] J. Malinzi, V.O. Juma, C.E. Madubueze, J. Mwaonanji, G.N. Nkem, E. Mwakilama, T. Mupedza, V. Nandwa, E.A. Bakare, I.L. Moyo-Zulu, E. Campillo-Funollet, T. Mazikana, S. Osman, A. Madzamuse and F. Nyabadza. Predicting the number of COVID-19 wave occurrences: Case of South Africa.. *GCRF-EPSRC UK-APASI program*, (2021).

****

Foot and Mouth Disease Transmission and Control in presence of environmental contamination

Kedibonye Molefe[a], **Notice Ringa**[b,c] & **John B. H. Njagarah**[a]
[a] *Department of Mathematics and Statistical Sciences,*
Botswana International University of Science and Technology,
Private Bag 16, Palapye, Botswana
K. Molefe: kedibonye.basi@studentmail.biust.ac.bw
J. Njagarah: njagarahh@biust.ac.bw
[b] *British Columbia Centre for Disease Control, Vancouver, BC, Canada*
[c] [c]*University of British Columbia, Vancouver, BC, Canada*
Email: notice.ringa@bccdc.ca

Abstract

In this work, a mathematical model for the dynamics of foot and mouth diseasa (FMD) accounting for the direct and indirect transmission routes is formulated and analysed. The model basic reproduction number ($\mathcal{R}_0$) is evaluated and clearly structured to highlight differential infectivity. Basic properties of the model such as existence and stability of the disease-free equilibrium as well as the existence of the endemic equilibrium were determined. Sensitivity analysis of $\mathcal{R}_0$ as well as the full model was performed using the Latin Hypercube Sampling scheme (LHS) for a selected set of input parameter values to determine the parameters that are significantly sensitive. Our sensitivity analysis showed that; on one hand the parameters describing direct and indirect transmission routes as well as the shedding of the virus into the environment by the infected animals, have the greatest positive Partial Rank Correlation Coefficients (PRCCs). On the other hand, the parameters describing the recovery rate of infected and carrier animals have the largest negative PRCCs. If the processes described by parameters with negative PRCCs are increased, then the chances of containing the disease are enhanced. Our numerical simulations shows that reducing the contact between susceptible animals and infected surfaces by 80% can reduce the disease burden by up to 71%. However, a combination of controls including reducing contact with infected surfaces and disinfecting which leads to increased decay of the pathogen by 100 folds from the baseline value can lead to elimination of the disease. From our results, we recommend that combinations of control measures should be implemented if the disease is to be contained in a shorter time as compared to piecemeal application of intervention measures.

Keywords: Foot and Mouth disease, Pathogen infested environment, Sensitivity analysis, combined intervention measures

Category: **Mathematical Biology**

References

[1] U. Wernery and J. Kinne. Foot and mouth disease and similar virus infections in camelids: a review. *Rev Sci Tech*, 31(3):907–918, 2012.

[2] S. M. Blower and H. Dowlatabadi. Sensitivity and uncertainty analysis of complex models of disease transmission: an HIV Model, as an Example. *International Statistical Review*, 62(2):229–243, 1994.

[3] C. Ray S. Marino, I. Hogue and D. Kirschner. methodology for performing global uncertainty and sensitivity analysis in systems biology. *J Theor Biol*, 254(1):178–196, 2008.

****

Mathematical modelling of dissolved oxygen in middle-sized fish ponds

José Nhavoto
Department of Mathematics and Informatics, Faculty of Science
Eduardo Mondlane University
jnhavoto@gmail.com

Abstract

A mathematical model was developed to predict the effects of wind speed, light, pH, Temperature, dissolved carbon dioxide and chemical oxygen demand (COD) on Dissolved Oxygen (DO) in a middle-sized fish ponds. Infrastructure consists of five concrete fish ponds with capacity for over 13.000 tilapia fingerlings. The effects of organic feeds, aeration and fish activity were added to the model developed by Kayombo et al. (2000) to reflect the situation in fish ponds. Model calibration and validation was done by use of average DO, pH, temperature, COD, CO2 and algae biomass data measured from fish ponds in Maputo; and light intensity data were adopted from [1]. Model results showed a linear relationship between simulated DO and measured DO in fish pond for model calibration and for model validation. Simulation results also showed a general decrease of DO with time in 13 days by 28 and 38% for first and second batch, respectively. Thus, the model developed in this study could be used to predict the DO dynamics in fish ponds. Based on the model results, successful cultivation of healthy fish may require that retention time for water in the fish pond be 10 days.

Keywords: Calibration, light, pH, substrate, temperature, validation

Category: **Mathematical Biology**

References

[1] S. Kayombo, T.S.A Mbwettea, A.W Mayoa, J.H.Y Katima, S.E. Jorgensen: Modelling diurnal variation of dissolved oxygen in waste stabilization ponds. *Ecological Modelling*, 127.1 (2000) (p. 21-31)

[2] Li, Wuyan; Wu, Hao; Zhu, Nanyang; Jiang, Yongnian; Tan, Jinglu; Guo, Ya: Prediction of dissolved oxygen in a fishery pond based on gated recurrent unit (GRU). *Information Processing in Agriculture* 8.1 (2021) (p. 185-193)

****

Dynamical survival analysis: from population to individual based epidemic models

Joseph Malinzi[a,b]
[a] Department of Mathematics, Faculty of Science and Engineering, University of Eswatini, Kwaluseni M201, Eswatini.
[b] Institute of Systems Science, Durban University of Technology, Durban 4000, South Africa.
jmalinzi@uniswa.sz

Abstract

Due to the need for mathematical tractability, population based epidemic models often ignore certain important characteristics of disease transmission patterns and the underlying populations. This often leads to inaccurate findings and poor forecasts [1].

In this work, we aim to firstly analyse the classical SIR epidemic model; which represents the transmission of populations from susceptible to infectious and recovered classes. The analysis involves calculating the model thresholds of the basic reproduction ratio, the maximum possible number of infectives and the number of people that eventually end up being infected. Next, the SIR population based epidemic model is transformed to a version that focuses on the fate of a single individual referred to as a survival dynamical system. I will, using the approaches in [1], delve into the derivation of how population-level dynamics imply probability laws for individual-level infection and recovery times which can be used for statistical inference. For illustration purposes, I shall use some epidemic data for both models.

Keywords: The SIR model, Dynamical survival analysis, Agent based modeling

Category: **Mathematical Biology**

References

[1] W.R. KhudaBukhsh, B. Choi, E. Kenah, and G.A. Rempała. (2020): Survival dynamical systems: individual-level survival analysis from population-level epidemic models. *Interface Focus*, 10.1 (2020): 20190048.

****

COVID-19 changing the face of the world. Can sub-Sahara Africa cope?

Mandidayingeyi Hellen Machingauta[a,b], **Bwayla Lungu**[c] & **Edward Lungu**[a]
[a] Department of Mathematics and Statistical Sciences, Botswana International University of Science and Technology, Palapye, Botswana.
hellen.machingauta@studentmail.biust.ac.bw

[b]Department of Surveying and Geomatics, Midlands State University, P bag 9055, Gweru, Zimbabwe.
pfupajenamh@msu.ac.zw

[c] Department of Food Science, University of California,Davis, CA 95616, USA.
blungu@ucdavis.edu

[a] Department of Mathematics and Statistical Sciences, Botswana International University of Science and Technology, Palapye, Botswana.
lungue@biust.ac.bw

Abstract

We formulate a mathematical model for the spread of the coronavirus which incorporates adherence to disease prevention. The major results of this study are: first, we determined optimal infection coefficients such that high levels of coronavirus transmission are prevented. Secondly, we have found that there exists several optimal pairs of removal rates, from the general population of asymptomatic and symptomatic infectives respectively that can protect hospital bed capacity and flatten the hospital admission curve. Of the many optimal strategies, this study recommends the pair that yields the least number of coronavirus related deaths. The results for South Africa, which is typical of other sub-Sahara African countries, show that failure to address hygiene and adherence issues will preclude the existence of an optimal strategy and could result in a more severe epidemic than the Italian COVID-19 epidemic.

Keywords: Hospital bed capacity, Removal rates, Optimal strategies

Category: **Mathematical Biology**

****

An Ebola virus disease model with fear and environmental transmission dynamics

Maureen Juga[a], **Farai Nyabadza**[b] & **Farai Chirove**[c]

[a] Department of Mathematics and Applied Mathematics, University of Johannesburg, Auckland Park Campus, Johannesburg, 2006, South Africa
maureenj@uj.ac.za

[b] Department of Mathematics and Applied Mathematics, University of Johannesburg, Auckland Park Campus, Johannesburg, 2006, South Africa
fnyabadza@uj.ac.za

[c] Department of Mathematics and Applied Mathematics, University of Johannesburg, Auckland Park Campus, Johannesburg, 2006, South Africa
fchirove@uj.ac.za

Abstract

Recent Ebola virus disease (EVD) outbreaks have been limited not only to the interactions between humans but also to the complex interplay of the environment, human and socio-economic factors. Changes in human behaviour as a result of fear can also affect disease transmission dynamics. In this paper, a compartmental model is used to study the dynamics of EVD incorporating fear and environmental transmission. We formulate a fear dependent contact rate function to measure the rate of person to person, as well as pathogen to person transmissions. The epidemic threshold and the model equilibria are determined and, their stabilities are analysed. The model is validated by fitting it to data from the 2019 and 2020 EVD outbreaks in the Democratic Republic of Congo. Our results suggest that the fear of death from EVD may reduce the transmission and aid the control of the disease, but it is not sufficient to eradicate the

disease. Policymakers need to also implement other control measures such as case finding, media campaigns, Quarantine and increase in the number of beds in the Ebola treatment centers, good laboratory services, safe burials and social mobilisation, to eradicate the disease.

Keywords: Ebola virus, fear, contaminated environment, reproduction number, pathogens.

Category: **Industiral Mathematics**

****

Mobility Based SEIAHR COVID-19 Pandemic Model: Case Study of Mozambique

Paulo Joaquim & **Sansão Pedro**
Faculty of Science, Eduardo Mondlane University, Mozambique
paulojoaquim323@gmail.com

Abstract

Since December 2019, the world has been experiencing one of the worst health crises caused by the COVID-19 pandemic. The rapid spatial expansion of covid leads us to conjecture that classical compartmental epidemiological models are limited because they do not consider the importance of reducing social connectivity and the importance of restricting mobility. Convinced, we carried out this research to understand how social connectivity and mobility affect the size of the pandemic. We introduced the parameter of social connectivity and the parameter of mobility in our epidemiological model SEIAHR and with this, we simulated in a road network of interprovincial passenger transport, considering the number of inhabitants per province, data collected in the 2020 Yearbook prepared by the National Institute of Statistic. With the simulation, we found that, limiting social connectivity reduces and delays the peak of the pandemic and, with mobility restriction at 10%, 20% and 30%, the total size of the pandemic until reaching the peak decreases by 92%, 85% and 75% respectively.

Keywords: Epidemic Modeling, SEIAHR, Pandemics, COVID-19.

Category: **Mathematical Biology**

****

Understanding the transmission pathways of Lassa Fever: a mathematical modelling approach

Praise-God Madueme[a] & **Faraimunashe Chirove**[b]
[a] Department of Mathematics and Applied Mathematics, University of Johannesburg
pmadueme@uj.ac.za

[b] Department of Mathematics and Applied Mathematics, University of Johannesburg
fchirove@uj.ac.za

Abstract

The spread of Lassa fever infection is increasing in West Africa over the last decade. The impact of this can better be understood when considering the various possible transmission routes. We designed a mathematical model for the epidemiology of Lassa Fever using a system of nonlinear ordinary differential equations to determine the effect of transmission pathways toward the infection progression in humans and rodents including those usually neglected. We analyzed the model and carried out numerical simulations to determine the impact of each of the transmission routes. Our results showed that the burden of Lassa fever infection is increased when all the transmission routes are incorporated and most single transmission routes are less harmful, but when in combination with other transmission routes, they increase the Lassa fever burden. It is therefore important to consider multiple transmission routes to better estimate the Lassa fever burden optimally and in turn determine control strategies targeted at the transmission pathways.

Keywords: Transmission, dynamics, *Mastomys*, Lassa fever

Category: **Mathematical Biology**

References

[1] Onah, I. S., Collins, O. C., Madueme, P.-G. U., and Mbah, G. C. E., (2020): Dynamical system analysis and optimal control measures of lassa fever disease model. *International Journal of Mathematics and Mathematical Sciences 2020. 2, 24*

[2] Akhmetzhanov, A. R., Asai, Y., and Nishiura, H. (2019): Quantifying the seasonal drivers of transmission for lassa fever in nigeria. *Philosophical Transactions of the Royal Society B 374(1775), 20180268. 1*

****

Modelling, analysis and simulation of a mechanobiochemical model for 2D and 3D cell migration

Richard Kagisho Mahlakwana[a], **Gerald Marewo**[b] & **Anotida Madzvamuse**[c]

[a] University of Limpopo, South Africa
richard.mahlakwana@ul.ac.za

[b] North-West University, South Africa
gtmarewo@gmail.com

[c] University of Sussex, United Kingdom
a.madzvamuse@sussex.ac.uk

Abstract

We present the development, analysis and numerical simulation of a mechanobiochemical model for 2D and 3D cell migration. The model couples biochemical reactions and biomechani-

cal forces. Force balance equation is used to model the mechanical properties for displacement, while the dynamics of the actin and myosin concentration are described by two reaction-diffusion equations. The moving grid finite element method is employed to obtain an approximate solution to the model system. The numerical results are supported by linear stability theoretical results close to bifurcation point during early stages of cell migration.

Keywords: Mechanobiochemical model, Force balance equation, Reaction-diffusion equation, Moving grid finite element method, Cell migration

Category: **Mathematical Biology**

****

Conditions for a Second Wave of COVID-19 Due to Interactions Between Disease Dynamics and Social Processes

Sansao A. Pedro[a], **Frank T. Ndjomatchoua**[b], **Peter Jentsch**[c,d], **Jean M. Tchuenche**[e], **Madhur Arand**[c] & **Chris T. Bauch**[d]

[a] Departamento de Matemática e Informática, Universidade Eduardo Mondlane, Maputo, Mozambique
sansaopedro@gmail.com/sansao.pedro@uem.ac.mz

[b] Sustainable Impact Platform, Geospatial Science and Modelling Cluster, International Rice Research Institute, Metro Manila, Philippines
author2email

[c] School of Environmental Sciences, University of Guelph, Guelph, ON, Canada
[d] Department of Applied Mathematics, University of Waterloo, Waterloo, ON, Canada,
[e] Avenir Health, Glastonbury, CT, United States.

Abstract

In May 2020, many jurisdictions around the world began lifting physical distancing restrictions against the spread of severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2). This gave rise to concerns about a possible second wave of coronavirus disease 2019 (COVID-19). These restrictions were imposed in response to the presence of COVID-19 in populations, usually with the broad support of affected populations. However, the lifting of restrictions is also a population response to the accumulating socio-economic impacts of restrictions, and lifting of restrictions is expected to increase the number of COVID-19 cases, in turn. This suggests that the COVID-19 pandemic exemplifies a coupled behavior-disease system where disease dynamics and social dynamics are locked in a mutual feedback loop. Here we develop a minimal mathematical model of the interaction between social support for school and workplace closure and the transmission dynamics of SARS-CoV-2. We find that a second wave of COVID-19 occurs across a broad range of plausible model input parameters governing epidemiological and social conditions, on account of instabilities generated by behavior-disease interactions. The second wave tends to have a higher peak than the first wave when the efficacy of restrictions is greater than 40% and when the basic reproduction number R_0 is less than 2.4. Surprisingly, we also found that a lower R_0 value makes a second wave more likely, on account of behavioral

feedback (although a lower R0 does not necessarily cause more infections, in total). We conclude that second waves of COVID-19 can be interpreted as the outcome of non-linear interactions between disease dynamics and social behavior. We also suggest that further development of mathematical models exploring behavior-disease interactions could help us better understand how social and epidemiological conditions together determine how pandemics unfold.

Keywords: COVID-19, epidemic model, behavioral fatigue, coupled behavior-disease system, SARS-CoV-2, evolutionary game theory, imitation dynamics, social learning

Category: **Mathematical Biology**

References

[1] He D, Dushoff J, Day T, Ma J, Earn DJ. Inferring the causes of the three waves of the 1918 influenza pandemic in England and Wales. *Proc R Soc B Biol Sci.* (2013) 280:20131345. doi: 10.1098/rspb.2013.1345

[2] Wells CR, Bauch CT. The impact of personal experiences with infection and vaccination on behaviour-incidence dynamics of seasonal influenza. Epidemics. (2012) 4:139–51. doi: 10.1016/j.epidem.2012.06.002

****

Modelling the Spatial Distribution of Possible Livestock Production Level using the SMAP soil moisture data: *The case of Botswana*

Semu Mitiku Kassa[a], **Amsalework Ayele Ejigu**[b] & **Manalebish Debalike Asfaw**[c]

[a] Department of Mathematics and Statistical sciences,
Botswana International University of Science and Technology, Palapye, Botswana
kassas@biust.ac.bw

[b] Climate data analysis division, Innovations in Development,
Education and the Mathematical Sciences (IDEMS) International, United Kingdom
amsalework16@gmail.com

[c] Department of Mathematics, Addis Ababa University, Ethiopia
manalebish.debalike@aau.edu.et

Abstract

In a traditional livestock farming, the livelihood of animals depends highly on existing plant biomass, which in turn is affected by the level and intensity of temperature, rainfall, humidity and other meteorological variables. Understanding the interaction of such meteorological factors and agricultural production in general is an important aspect in planning at the macro and micro levels. Particularly livestock agriculture is heavily affected by the changing climate, and hence the variation in major meteorological variables. However, there is still limited research regarding the impacts of meteorological variables on livestock production in each particular

region. Soil moisture is one of the main factors in agricultural production and hydrological cycles with better memory of previous weather conditions. It also involves complex structural characteristics and meteorological factors. In this study, a soil moisture dependent mathematical model for the interaction of plants and herbivores is developed and analysed. The Soil Moisture Active Passive level 4 satellite soil moisture data is used in the model to simulate the possible spatial distribution of plants and the corresponding potential livestock production level for Botswana. A global dynamic sensitivity analysis is employed to study the sensitivity of the solution of the model with a variation in the involved parameter values. The results of the simulations of the model show that estimated livestock harvest in wet regions is more than triple as compared to what is estimated for dry regions. If some important parameters are properly estimated and the soil moisture data is available for each region, it is possible to estimate using the proposed model the livestock production level for each spatial region with better accuracy.

Keywords: Ecological model; Plant–herbivore interaction dynamics; Non-constant harvest; Soil moisture; Livestock production; Spatial distribution

Category: **Mathematical Biology**

References

[1] M. D. Asfaw, S. M. Kassa, E. M. Lungu and W. Bewket (2019): Effects of temperature and rainfall in plant-herbivore interactions at different altitude, *Ecological Modelling* 406 : 50 – 59.

****

Human papillomavirus prevention: vaccination as strategy for controlling the disease and the existence of a dual-rate effect

Simphiwe Simelane[a], & **Phumlani Dlamini**[b]
[a] University of Johannesburg, South Africa
ssimelane@uj.ac.za

[b] University of Johannesburg, South Africa
pgdlamini@uj.ac.za

Abstract

Human papillomavirus (HPV) is one of the most common sexually transmitted diseases and is a major cause of cervical cancer. Even though there are numerous intervention strategies to minimize the spread of HPV, it is reported that more than 80% of the sexual active population in the world has HPV. We develop a simple HPV model to investigate vaccination strategies to control the disease. We consider a continuous vaccination strategy where only the susceptible are vaccinated. We investigate and present equilibrium quantities as well as conditions for them to exist. The impact of vaccination on overall disease dynamics is discussed through simple numerical simulations. Results indicate that vaccination could be targeted by public health

officials to reduce disease burden in communities. We also investigate the existence of a dual-rate effect and also determine the conditions under which it occurs. In the presence of such an effect, an initial large scale vaccination drive (vaccines with high efficacy) may reduce the disease prevalence to a level that can be effectively controlled at a relatively lower vaccination rate and a relatively affordable cost.

Keywords: Human papillomavirus virus, Vaccination rate, Dual-rate effect, Numerical simulations.

Category: **Mathematical Biology**

References

[1] Saldana, F., Korobeinikov, A., and Barradas, I. (2019): Optimal control against the human papillomavirus: protection versus eradication of the infection. *Abstr. Appl. Anal., 1-13*.

[2] World Health Organization: Global strategy to accelerate the elimination of cervical cancer as a public health problem. https://www.who. int/publications/i/item/9789240014107. Accessed 05 March 2022.

[3] Genital HPV infection: CDC fact sheet. Centers for Disease Control and Prevention. http://www.cdc.gov/std/HPV/STDFact-HPV.htm. Accessed 11 June 2022.

[4] Oduro, B. and Magagula, V. M. (2021): COVID-19 intervention models: An initial aggressive treatment strategy for controlling the infection. *Infectious Disease Modelling, 6, 351-361*.

****

Modelling the Potential Impact of Stigma on the Transmission Dynamics of COVID-19 in South Africa

Siphokazi Gatyeni[a], **Faraimunashe Chirove**[b] & **Farai Nyabadza**[c]

[a] Mathematics and Applied Mathematics Department, University of Johannesburg, Johannesburg 2092, South Africa
siphokazig@uj.ac.za

[b] Mathematics and Applied Mathematics Department, University of Johannesburg, Johannesburg 2092, South Africa
fchirove@uj.ac.za

[c] Mathematics and Applied Mathematics Department, University of Johannesburg, Johannesburg 2092, South Africa
fnyabadza@uj.ac.za

Abstract

The COVID-19 pandemic continues to be a problem in South Africa. Individuals affected and infected by the disease suffer from stigma resulting in increased COVID-19 infections. In this paper, we developed a mathematical model to assess the effects of stigma on COVID-19 in South Africa, using low, moderate, and high stigma regimes in the population. The mathematical model was analysed and the basic reproduction number, R_0, of the COVID-19 model with stigma was determined. The model was then fitted to data of the four COVID-19 waves for the new daily infected cases, and the estimated parameter values from different waves are presented. The effects of stigma on COVID-19 waves were examined using the four stigma regimes (high, moderate, low, and stigma-free regimes). Our results revealed that stigma is instrumental in the increase in the number of COVID-19 infections. It is also a significant contributor to sustaining COVID-19 in the population and probably in other infectious diseases such as HIV/AIDS and sexually transmitted diseases. The results obtained can influence policy directions with respect to stigma and its impact on the transmission dynamics of diseases.

Keywords: COVID-19, stigma, modelling, global stability, basic reproduction number, sensitivity analysis

Category: **Mathematical Biology**

References

[1] SA Coronavirus. About COVID-19 (Coronavirus). Available online: https://sacoronavirus.co.za/information-about-the-virus-2/ (accessed on 27 February 2021).

[2] Sotgiu, G.; Dobler, C.C. Social stigma in the time of Coronavirus. European Respiratory Journal. 2020, 56, 2002461.

****

Modelling the interplay between police and criminals: a population dynamics approach

Tichaona Chikore[a], **Jane K. A. White**[b] & **Farai Nyabadza**[c]

[a] University of Johannesburg, South Africa
tichaona.chikore@gmail.com

[b]Bath University, UK
maskajw@bath.ac.uk

[c] University of Johannesburg, South Africa
fnyabadza@uj.ac.za

Abstract

The interplay between police and criminals is key to the management and control of crime. This paper presents a study on the interaction between criminals and the police using theory on the dynamics of exploited populations. The compartmentalized investigation of the effect of imposing effective baseline policing on the criminals' propensity to commit crimes uses a system of two non-linear ordinary differential equations of the Lotkka-Volterra type. It is a comparison of two recruitment strategies; one influenced by criminal abundance versus an independent strategy. The steady-states are determined and analyzed. Model predictions reveal backward bifurcations due to a *policing efficiency*, Π_0, which vanishes if there is no baseline police population or if policing fails to remove any criminals. Consequently, we get the minimum level of policing efficiency, above which there are no endemic steady-states. We validate this model using both estimated and recorded London homicide data between the years 2010 and 2019. We obtain baseline police thresholds necessary for the stability of the endemic steady-states, to maintain a sustainable co-existence of the police and criminals population, and to eventually eliminate the criminals in society. These results have important implications on how police and criminals interact, part of the fundamentals in the designing and implementation of containment strategies in the fight against crime.

Keywords: Mathematical model; Criminal activity; Police effort; Policing efficiency; Stability; Simulations.

Category: **Mathematical Biology**

****

Computer simulation of the dynamics of a spatial SIR epidemic model with time delays in transmission and treatment

Tiomela Tanefo Sedrique[a]

[a] University of yaounde 1 (Cameroon)

sedriquephd2020@gmail.com

Abstract

In human history, the spread of infectious diseases has repeatedly leads to epidemics, which could cause a devastating disaster to the earth and mankind. when such diseases increase in incidence and tend to spread geographically within decades, they can be defined as emerging infections. Diseases that reappear after a significant decline are called reemerging diseases. Generally, the appearance of these diseases always causes sanitary and economic crisis globally leading to planetary fluc- tuations, particulary in the low income countries like those in Africa. In this work, we analyze the spatial-temporal dynamics of a susceptible-infected-recovered (SIR) epidemic model with time de- lays. To better describe the dynamical behavior of the model, we take into account the cumulative effects of diffusion in the population dynamics, and the time delays in both the Holling type II treatment and the disease transmission process, respectively. To this end, we perform linear stability analyses at the disease-free and endemic equilibria, provide the expression of the basic reproduction number and set conditions on the

backward bifurcation using Castillo's theorem. The values of the critical transmission time delay, the treatment delays and the relationship between them are established. As results, we show that the transmission and treatment time delays are inversely proportional to the susceptible and infected diffusion rates. The analytical results are numerically tested. The results show how the treatment rate significantly reduces the density of infected population and ensures the transition from the unstable to the stable domain. Moreover, the system is more sensible to the treatment in the stable domain. As conclusion, the density of infected population increases with respect to the infected and susceptible diffusion rates. Both effects of treatment and transmission delays significantly affect the behavior of the system. The transmission time-delay at the critical point ensures the transition from the stable (low density) to the unstable (high density) domain.

Keywords: SIR model, Spatial diffusion, Transmission and treatment delay, Basic reproduction number, Nonlinear bifurcation analys.

Category: **Epidemiological modeling, Numerical Approximation of solutions of PDEs, Data Science, Mathematical Biology.**

References

[1] Tiomela, S.A.et al: Computer simulation of the dynamics of a spatial susceptible-infected-recovered epidemic model with time delays in transmission and treatment. Computer Methods and Programs in Biomedicine 212, 106469 (2021)

****

11 Mathematical Education

Teaching and Learning Mathematics and Physics with free ICT Tools: Case of GeoGebra.

Justino M. Buanali[a], **Silva J. Muhate**[b], **Lucas J. Boene**[c], & **Felix Banze**[d]
[a,b,c,d] Instituto Superior Politécnico de Songo
[a] jbuanali@gmail.com
[b] silvamuhate@gmail.com
[c] lucasboene@gmail.com
[d] felixbanze@gmail.com

Abstract

According to UNESCO, Information and Communication Technology (ICT) can contribute to universal access to education, equity in education, the delivery of quality learning and teaching, teachers' professional development and more efficient education management. The continuous technological development has provided the transfer of several technologies for application in the teaching and learning process. For the context of STEM, several software's have been

created to help in the process of visualization and modeling of different topics. GeoGebra, a free dynamic software for all grade levels, combines tools from geometry, algebra, statistics, calculus and more, in a learning environment that is very interactive and collaborative between student and instructor. Through an exhaustive review in the literature, this article explores several applications and current trends of the use of GeoGebra. The paper reviews more than 24 articles covering various concepts and applications of GeoGebra in the areas highlighted above. The conclusions of most of the studies point to the positive effects of GeoGebra on the efficiency and effectiveness of teaching and learning of topics related to STEM. Therefore, the exercise allowed us to combine studies by different authors from different sides of the world and conclude that the use of GeoGebra software can help in the understanding and development of autonomous learning, through visualization and modeling, allowing students to create or increase interest in Mathematics and Physics topics.

Keywords: GeoGebra, Teaching and Learning, Visualization, STEM, Literature Review

Category: **Mathematics Education**

References

[1] Zatdinov, R. and Valles, J. R. Jr. (2022):Synthesis of Modeling, Vizualization and Programming in GeoGebra as an effective approach for Teaching and Learning STEM Topics.

[2] Solvang, L. and Haglund, J. (2021): How can GeoGebra support Physics education in upper-secondary school (a review).

[3] Filho, M de A. B., Filho R. S. de M. C. and Amaral, F. M. (2022): O uso da modelagem matemática com o GeoGebra no ensino de funções trigométricas: Uma revisão bibliográfica.

****

Relative position of lines in the space: the computer between what the eyes and the mind see

Paulo Diniz
Universidade Licungo, Beira, Mozambique
padibene2@yahoo.com.br

Abstract

The present work aims to describe relevant aspects of a teaching experience with undergraduate students in Mathematics Teaching, at Universidade Licungo, in Mozambique. This teaching experience has been motivated by the low performance of the students when they solved a written test in the subject of Analytical Geometry. With a focus on one of the questions that made up the written test, it was necessary to answer the following question: How to discuss the resolution of the mathematical task exposed to students so that they understand the rationality

of the production of their solution? To answer this question, it was necessary to carry out a teaching experiment, in the classroom, where students were observed in mathematical activity and triggered teacherstudent interaction processes. The results showed that students had difficulties in relating the texts represented by the drawings they made in their notebooks and the texts emerging from the algebraic elaboration of the solutions. With this, it was evident the computer's contribution to dispel apparent inconsistencies between the results obtained through paper drawings and vector analysis.

Keywords: Geometry,Visualizartion, Computer, Line

Category: **Mathematics Education**

****

Language in algebraic thinking: case of second language learners in Mozambique

Ribas Guambe
Department of Mathematics and Informatics, Universidade Eduardo Mondlane, Maputo, Mozambique
ribas.guambe@uem.ac.mz

Abstract

Mozambique is a multilingual society, with around 40 native languages of Bantu origin, a part of the Portuguese language, the main and official language of administration, business and instruction. As pointed out in some studies, proficiency in the language of instruction is generally seen as linked to performance in mathematics at school and that, in the situation where the language of instruction is different from that spoken in everyday life, there is a much larger "gap", particularly when it comes to mathematics teaching and learning processes than in other teaching-learning contexts. Regarding the situation in Mozambique, the lack of development of specialized mathematical records in native languages is visible, since, even in simple commercial relationships, people mark and discuss prices and carry out counting and basic operations mostly in Portuguese. The study explored grade 8 students' use of algebraic symbols, the role of language in algebraic thinking and students' understanding of mathematical language, and. Data collection involved 8 students selected from an 8th grade class at a semi-rural school in Maputo Province, southern Mozambique. Data collection consisted of written test items and interviews. Data analysis showed that learners struggled to discover the meaning of letters or simply associated them with numbers according to their position in the alphabetic system. Students struggled to establish the correct meaning of words or terms written in common language in math language and struggled to find what they were supposed to find or do in each verbal algebra problem due to their misunderstanding of math language. The study results in the argument that the language difficulties that second language students face in algebraic thinking are not only related to their limited fluency in the learning context, but also to the inability of students to move from spoken mathematical language to mathematical language writing.

Keywords: algebraic language, word problems, learning.

Category: **Mathematics Education**

****

12 MATHEMATICS FOR INDUSTRY

Conservation laws and exact solutions of a coupled variable-coefficient modified Korteweg–de Vries system

Abdullahi Adem[a,b]
[c] Department of Mathematical Sciences, University of South Africa, UNISA, Pretoria 0003, South Africa
ademar@unisa.ac.za

Abstract

In this talk we study a generalized coupled variable-coefficient modified Korteweg–de Vries (CVCmKdV) system that models a two-layer fluid, which is applied to investigate the atmospheric and oceanic phenomena such as the atmospheric blockings, interactions between the atmosphere and ocean, oceanic circulations and hurricanes. The conservation laws of the CVCmKdV system are derived using the multiplier approach and a new conservation theorem. In addition to this, a similarity reduction and exact solutions with the aid of symbolic computation are computed.

Keywords: generalized coupled variable-coefficient modified Korteweg–de Vries system, symbolic computation, conservation laws, similarity reductions, exact solutions.

Category: **Industiral Mathematics**

****

A New One-Dimensional Finite Volume Method for Hyperbolic Conservation Laws

José Pedro[a], **Mapundi Banda**[b] & **Precious Sibanda**[c]
[a]Department of Engineering and Technology, Instituto Superior Politécnico de Tecnologias e Ciências-ISPTEC, Luanda, Angola
jose.caluyna@isptec.co.ao

[b] Department of Mathematics and Applied Mathematics, University of Pretoria, Private Bag X20, Hatfield 0028, South Africa
mapundi.banda@up.ac.za

[c] School of Mathematical Sciences, University of KwaZulu-Natal, Private Bag X01, Pietermaritzburg Campus, Scottsville 3209, South Africa
sibandap@ukzn.ac.za

Abstract

In this paper, a new one-dimensional Finite Volume Method for Hyperbolic Conservation Laws is presented. The method consists of an improved numerical inter-cell flux function at the element interface. To back theoretically the method, necessary components for convergence are presented. Therefore, it is proved that the method is consistent with the P.D.E and that it is monotone with respect to its variables. Moreover, to validate the approach and show its efficiency, we compute several one-dimensional test problems with discontinuous solutions and we make comparisons with traditional methods. The results show an improvement in the non-oscillatory shock-capturing properties based on the new approach.

Keywords: Finite Volume Method, Numerical Flux, Conservation Laws, Non-Oscillatory Approach, Method Exactly Conservative

Category: **Industiral Mathematics**

References

[1] Dumbser, M. and Käser, M. (2007): Arbitrary high order non-oscillatory finite volume schemes on unstructured meshes for linear hyperbolic *Journal of Computational Physics.*, 221:693–723, 2007.

[2] Pedro, J. C., Banda, M. K. and Sibanda, P. (2014): On one-dimensional arbitrary high-order WENO schemes for systems of hyperbolic conservation laws *Journal of Computational & Applied Mathematics.*, 33:363–384, 2014.

[3] Pedro, J. C., Banda, M. K. and Sibanda, P. (2020): A New One-Dimensional Finite Volume Method for Hyperbolic Conservation Laws *Journal of Applied & Computational Mathematics.*, 9:1, 2020.

[4] Sun, X., Wang, G. and Ma., Y. (2020): A New Modified Local Lax-Friedrichs Scheme for Scalar Conservation Laws With Discontinuous Flux *Applied Mathematics Letters.*, 105:106328, 2020.

****

The grazing bifurcations and transitions between periodic solutions in a periodically forced PP04 model for glacial cycles

Kgomotso S. Morupisi[a] & **Chris Budd**[b] [a] Botswana International University of Science and Technology
morupisik@biust.ac.bw

[b] University of Bath
mascjb@bath.ac.uk

Abstract

We look at the periodic behaviour of the Earth's glacial cycles and the transitions between different periodic states when either external parameters (such as ω) or internal parameters (such as d) are varied. We model this using the PP04 model of climate change. This is a forced discontinuous Filippov (non-smooth) dynamical system. When periodically forced this has coexisting periodic orbits. We find that the transitions in this system are mainly due to grazing events, leading to grazing bifurcations. An analysis of the grazing bifurcations is given and the impact of these on the domains of attraction and regions of existence of the periodic orbits is determined under various changes in the parameters of the system. Grazing transitions arise for general variations in the parameters (both internal and external) of the PP04 model. We find that the grazing transitions between the period orbits resemble those of the Mid-Pleistocene.

Keywords: Dynamical System, Filippov system,Non-smooth system, Grazing Bifurcation, Basin of Attraction, Relaxation Oscillator.

Category: **Industiral Mathematics**

References

[1] Bernardo, M., Budd, C., Champneys, A.R. and Kowalczyk, P.(2008):Piecewise-smooth dynamical systems: theory and applications, vol 163, Springer Science & Business Media

[2] Paillard, D. and Parrenin, F.(2004): The Antarctic ice sheet and the triggering of deglaciations,*Earth and Planetary Science Letters*,vol 227,No. 3-4,pages 263-271,Elsevier

[3] Morupisi, K.S and Budd, C. (2021): An analysis of the periodically forced PP04 climate model, using the theory of non-smooth dynamical systems,*IMA Journal of Applied Mathematics*,vol 86,No 1,pages 76-120,Oxford University Press

[4] Filippov, A. F.(2013):Differential equations with discontinuous righthand sides: control systems,vol 18,Springer Science & Business Media

[5] Petit, J., Jouzel, J., Raynaud, D. and Barkov, Narcisse I., Barnola, J-M., Basile, I., Bender, M.,Chappellaz, J., Davis, M., Delaygue, G. (1999): Climate and atmospheric history of the past 420,000 years from the Vostok ice core, Antarctica,*Nature*,vol 399, No.6735, pages 429-436, Nature Publishing Group

****

Machine learning models for flood mapping and damage assessment in Mozambique

Manuel Nhangumbe[a], **Andrea Nascetti**[b] & **Yifang Ban**[b]

[a] Dep. of Mathematics and Computer Science, Faculty of Science, Eduardo Mondlane Unversity, Maputo

manuel.anhangumbe@uem.mz

[b] Division of GeoInformatics, Department of Urban Planning and Environment, KTH-Royal Institute of Technology, Stockholm, Sweden

nascetti@kth.se

Abstract

Floods have increased in frequency and intensity in recent years and have been linked to climate change among other causes. Moreover, they occur in an increasingly diverse set of locations and bring massive destruction of infrastructures, devastation of agricultural fields, and human loss and displacement. The event causes food insecurity, creates or increases poverty in the affected populations, and in some cases, largely contributes to the emergence of water borne diseases. Mozambique is considered the most prone country to flooding in Southern Africa. In the last years, the frequency of tropical cyclones that come with heavy rains has increased (Cyclone Idai on 15th March 2019, Cyclone Kenneth on 25th April 2019, Cyclone Chalane on 30th December 2020, Cyclone Eloise on 23rd January 2021, Cyclone Guambe on 19th February 2021) creating considerable damages on infrastructures, human displacement, and devastation on agricultural fields. Cyclone Idai, for instance, destroyed 93% of the city of Beira (4th largest city in Mozambique), the capital of Sofala province in the central part of the country [1, 2]. Covering large areas at regular revisits, satellite remote sensing is playing an important role in disaster management such as floods, fire, cyclones, and earthquake, especially for preparedness, warnings, and emergency response.

In Mozambique, several studies have been conducted in collaboration with the National Institute for Disaster Management (locally called INGD) to analyze the risks of floods and mitigate the impact, including creating some algorithms for mapping flooded areas using drones. However, cost-effective large-scale mapping and damage assessment methods are a high priority [3]. With the launches of Sentinel-1 and Sentinel-2, free and open data with global coverage, large swath and high temporal resolution became routinely available.

In this research we aimed to investigate multitemporal Sentinel-1 SAR (10m resolution), Sentinel-2 MSI (10m resolution). We investigated the use of a freely available benchmark dataset released in 2021 for a code challenge competition. This data is based on Sentinel-1 SAR imagery obtained from a global database Cloud to Street that consists of 18 flood events and provides high detailed reference maps of 13 countries in Africa, America, Europe and Asia. We then compared supervised and unsupervised machine learning methods on uni-temporal Sentinel-1 SAR imagery using this dataset, splitting the reference maps into training and validation sets. We performed the classification exploiting the capabilities of Google Earth Engine (GEE) platform. The data is ingested into GEE, and since it comprises of stacks of VH and VV Sentinel-1 flood imagery, we computed a pixel-based classification combining VH and VV backscattering information. After the classification we applied a Gaussian filter to reduce the noise in the flood mask improving the overall accuracy. We then repeat the procedure adding Sentinel-1 flooding data of cyclone Idai. To compute the accuracy, we compare our results with the validation data by checking how close our method detects floods compared to validation data. Moreover, we investigated the use of Sentinel-2 MSI to produce a land cover map of the study area and estimate the percentage of flooded areas for each land cover class. To leverage the damage assessment quality of the results we integrated population data from Beira. The results show that the combination of Sentinel-1 SAR and Sentinel-2 MSI data is promising for near real-time flood mapping and damage assessment. Using the imagery acquired on 19th March 2019 in Beira by Sentinel-1 SAR, it was possible to automatically map flooded areas with intersection over the union metric of 0.56. We plan to share the method and the developed code as free and open-source software to support future research on this topic and promote the use of this data during the emergency as support cartographical information.

Keywords: Sentinels 1 and 2, Flood mapping, Machine learning, Classification, Damage As-

sessment.

Category: **Machine Learning and Artificial Intelligence**

References

[1] WHO and INS. (2019): Tropical cyclones idai and kenneth, Mozambique: *National Situation Report 7*.https://www.afro.who.int/sites/default/files/2019-08/National%20SitRep%207_MOZ_1%20%20to%2014%20Jul%202019_ENG.pdf.

[2] OCHA (2022): Cyclones idai and kenneth. https://www.unocha.org/southern-and-eastern-africa-rosea/cyclones-idai-and-kenneth. *[Online; accessed on Septembe 27, 2022].*

[3] Antoine, R. and Lopez, T. and Tanguy, M. and Lissak, C. and Gailler, L. and Labazuy, P. and Fauchard, C. (2019): Geoscientists in the sky: Unmanned aerial vehicles responding to geohazards. *Surveys in Geophysics, 41(6):1285–1321*.

****

On the symbolic computation of exact solutions and conservation laws of a generalized (2+1)-dimensional Calogaro-Bogoyavleskii-Schiff equation

Motshidisi Charity Sebogodi[a], **Ben Muatjetjeja**[a,b] & **Abdullahi Rashid Adem**[c]

[a] University of South Africa, UNISA, South Africa
charitymoroke@gmail.com

[b] Department of Mathematics, Faculty of Science, University of Botswana, Private Bag 22, Gaborone, Botswana
muatjetjejab@ub.ac.bw

[c] Department of Mathematical Sciences, University of South Africa, UNISA, Pretoria 0003, South Africa
ademar@unisa.ac.za

Abstract

New exact solutions of the (2+1)-dimensional Calogaro-Bogoyavlenskii-Schiff equation are presented. This equation was first erected by Bogoyavlenskii and Schiff in unique ways, namely Bogoyavlenskii used the modified Lax formalism, whereas Schiff employed the same equation by reducing the self-dual Yang Mills equation. We also derive conserved vectors of the above mention equation by invoking the multiplier method.

Keywords: Exact solutions; Lie point symmetries; Conservation laws.

Category: **Industiral Mathematics**

References

[1] G.M. Moatimid, R.M. EI-Shiekh, A.A.A.H. AI-Nowehy, Exact solutions for Calogero-Bogoyavlenskii-Schiff equation using symmetry method, Appl. Math. Comput. 220 (2013) 455-462

[2] Muatjetjeja, B. Coupled Lane-Emden-Klein-Gordon-Fock system with central symmetry: Symmetries and Conservation laws. *J. Differ. Equ. 263 (2017) 8322–8328.*

[3] Freire, I.L. and Muatjetjeja, B. Symmetry analysis of a Lane-Emden-Klein-Gordon-Fock system with central symmetry. *Discrete. Contin. Dyn Syst. Ser A. 11 (2018) 667–673.*

****

Multistage Optimization Algorithm Using Chaotic Search

Tayeb HAMAIZIA[a]

[a] Department of Mathematics, Faculty of Sciences Exacts, University Constantine 1 Algeria

h2tayeb@gmail.com

Abstract

The chaotic optimization method gets increasing attention as an engineering application of the chaotic dynamical systems. In this poster we undertake a performance analysis for a new class of evolutionary algorithms called chaos optimization algorithm (COA), recently proposed by Caponetto and al. [1], [2], [3], It was originally proposed to solve nonlinear optimization problems with bounded variables. Different chaotic mapping have been considered, combined with several working strategy. In this work, a chaotic strategy is proposed based on a two-dimensional discrete chaotic attractor. Experiments results showed that the proposed algorithm can achieve good performance. **Keywords:** global optimization, chaos search, function test

Category: **Fluid Dynamics**

References

[1] R. Caponetto, L. Fortuna, S. Fazzino and M. G. Xibilia, Chaotic Sequences to Improve the Performance of Evolutionary Algorithms, IEEE Transactions on Evolutionary Computation, Vol.7,n3,June2003,289-304.

[2] T. Hamaizia, R. Lozi, An improved chaotic optimization algorithm using a new global locally averaged strategy, Journal of Nonlinear Systems and Applications 3 (2) (2012) 58-63.

[3] L. S. Coelho, Tuning of PID controller for an automatic regulator voltage system using chaotic optimization approach, Chaos, Solitons and Fractals, 39, 2009, 1504-1514.

****

On the solutions of a (3+1)-dimensional novel KP-like equation

T.S. Moretlo[a,b] **B. Muatjetjeja**[a,c] **A.R. Adem**[d]

[a] Department of Mathematical Sciences, North-West University
Private Bag X 2046, Mmabatho 2735, Republic of South Africa

[b] Department of Mathematical Sciences, Sol Plaatje University,
Central Campus, Private Bag X5008, Kimberly 8300,
Republic of South Africa

[c] Department of Mathematics, Faculty of Science, University of Botswana
Private Bag 22, Gaborone, Botswana

[d] Department of Mathematical Sciences, University of South Africa
UNISA 0003, Republic of South Africa

Email: thabo.moretlo@spu.ac.za muatjetjejab@ub.ac.bw ademar@unisa.ac.za

Abstract

This talk aims to construct exact solutions of a new (3+1)-dimensional novel KP-like equation, which arises in the analysis of versions of resonant phenomena. The classical symmetry approach will be employed to search for exact solutions of a newly nonlinear evolution equation(NLEE). Thereafter, we search for the admitted conserved vectors of the mentioned novel KP-like equation.

Keywords: Lie point symmetries; Symmetry reduction; Exact solutions; Conservation laws

Category: **Industrial Mathematics**

References

[1] R. Hirota, Exact solution of the Korteweg-de Vries equation for multiple collisions of solitons, *Physical Review Letters*, 27 (1971) 1192-1194.

[2] S.K. Liu, Z.T. Fu, S.D. Liu, Q. Zhao, Jacobi elliptic function expansion method and periodic wave solutions of nonlinear wave equations, Phys. Lett. A. 289 (2001) 69-74.

[3] C. Gu, H. Hu, Z. Zhou, Darboux Transformation in Soliton Theory and Its Geometric Applications, Springer, The Netherlands, 2005.

[4] M. Wazwaz, The Tanh and Sine-Cosine Method for Compact and Noncompact Solutions of Nonlinear Klein Gordon Equation, Appl. Math. Comput. 167 (2005) 1179-1195.

[5] G.W. Bluman and S. Kumei, Symmetries and Differential Equations, Applied Mathematical Sciences, 81, Springer-Verlag, New York, 1989.

****

Conservation laws and exact solutions of a generalized (3+1)-dimensional nonlinear wave in liquid with gas bubbles

Thebeetsile Jackson Podile[a,c], **Ben Muatjetjeja**[b,c] & **Abdullahi Rashid Adem**[c]
[a] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
podiletj@tut.ac.za

[b] Department of Mathematics, Faculty of Science, University of Botswana, Private Bag 22, Gaborone, Botswana
muatjetjejab@ub.ac.bw

[c] Department of Mathematical Sciences, University of South Africa, UNISA, Pretoria 0003, South Africa
ademar@unisa.ac.za

Abstract

In this talk we consider a (3+1)-dimensional nonlinear wave equation, which decribes nonlinear phenomena in liquid containing gas bubbles. In the analysis, we use the Lie symmetry method. Moreover, exact solutions for the (3+1)-dimensional nonlinear wave are obtained using Kudryashov method and Ricatti equation method. Furthermore, conservation laws are computed using the multiplier method.

Keywords: A (2+1)-dimensional Korteweg-de Vries type equation in water waves; Lie symmetry analysis; Multiple exp-function method; Conservation laws.

Category: **Industiral Mathematics**

References

[1] G.W. Bluman and S. Kumei, Symmetries and Differential Equations, Applied Mathematical Sciences, 81, Springer-Verlag, New York, 1989.

[2] P.J. Olver, Applications of Lie Groups to Differential Equations, Graduate Texts in Mathematics, 107, 2nd edition, Springer-Verlag, Berlin, 1993.

[3] N.H. Ibragimov, CRC Handbook of Lie Group Analysis of Differential Equations, Vol 1-3, Ibrahimov, N.H. ed., CRC Press, Boca Raton, Florida, 1994-1996.

[4] A. M. Wazwaz, A study on the (2+1)-dimensional KdV4 equation derived by using the KdV recursion operator. *Mathematical Methods in the Applied Sciences*, 36 (2013), 1760–1767.

****

13 THEORETICAL PHYSICS

Spatio-temporal variability of short-scale solar energy availability in the southern region of Mozambique

Fernando Mucomole[a], **Carlos Silva**[b], & **Lourenço Magaia**[c]
[a] Eduardo Mondlane University, Faculty of Sciences – Department of Physics, Mozambique,
doctoramentofvm@gmail.com
;
[b] University of Lisbon, Instituto Superior Técnico – Department of Mechanical Engineering, Portugal, Lisbon,
carlos.santos.silva@tecnico.ulisboa.pt
[c] Eduardo Mondlane University, Faculty of Science – Department of Mathematics and Informatics, Mozambique,
lourenco.l.magaia@uem.ac.mz

Abstract

The needs for energy use have been increasing all over the world, in Mozambique the installed solar photovoltaic capacity increased from $2000GW_p$ to more than $4000GW_p$ between the years 2014 to 2022. Motivated by the need to establish an environment of efficient projection of photovoltaic systems, we aimed to quantify the spatio-temporal variability of the availability of solar energy in a short measurement scale. The theoretical-descriptive method was used, which consists of the spatio-temporal analysis of the clear-sky index, having taken as materials: the sample of solar radiation data from the southern region of Mozambique, in the provinces of Maputo, in Lagoa Phati, on the campus from Eduardo Mondlane University and a station in Gaza, in Dindiza during the years 2012, 2013 and 2014. First, the data were processed using the theory of errors, selecting the types of days from the calculation of the clear-sky index K_t^* for temporal analysis, then the correlation was made for spatial variability analysis and using Python histograms of the types of days were modeled and the variability rate determined. The results show that of the total of about 7 years processed, about seventy months (usable area), distributed in all seasons of the year, the southern region of Mozambique presents mostly with clear sky days, in detriment of intermediate days. relating to cloudy days. From the statistical analysis of the frequency density variability it is shown that the variation of $K_t^*(\delta K_t^*)$ has a maximum close to zero, and it gradually decreases as they increase in value. The intermediate sky days have a similar behavior to the previous ones, however it presents a slight decrease, because for δK_t^* in the interval $[-2, 2]$ it is higher. These results agree with those reported in the literature by Lohmann et. al.,(2016). It can be concluded that the values of K_t^* for a day vary between $0.3342 - 1.2764$, with the minimum observed in July and the maximum in December and the variations during the daily course of the K_t^* index determined according to their standard deviation show so much adequacy to the adopted model.

Keywords: Spatial, temporal, clear sky, variability, intermediate

Category: **Theoretical Physics**

References

[1] Duffie, A. John & Beckman, A. William (1991), Solar Engineering of Thermal Processes;

[2] Lohmann, Gerald M. (2018), Irradiance Variability Quantification and Small-Scale Averaging in Space and Time: A Short Review.

****

A numerical solver for the Lane-Emden equation

Gerald Marewo[a],

[a] North-West University, Private Bag X6001, Potcheffstroom, 2520, Republic of South Africa

gerald.marewo@nwu.ac.za

Abstract

In this study we consider a boundary value problem for the Lane-Emden equation which describes the behaviour of the density of a gas sphere in hydrostatic equilibrium. The Lane-Emden equation is an example of the non-linear Poisson equation. The non-linearity of the equation motivates the use of a numerical solver. Our choice of the solver is a Galerkin finite element method coupled with linearisation. Finite element methods make a good choice because they handle complex geometries well and they have well developed techniques for their mathematical analyses among other desirable properties. The numerical solver is implemented on the computer using `MATLAB`, a computer environment for performing numerical computations and visualisation. Numerical experiments are performed to show that the solver is computationally effective.

Keywords: Lane-Emden equation, Galerkin finite element method, Linearisation, `MATLAB`

Category: **Mathematics for Industry**

References

[1] Zhao-xiang Li and Zhong-qing Wang, Pseudospectral methods for computing the multiple solutions of the Lane-Emden equation, *Journal of Computational Physics* **255** (2013) 407-421.

[2] S. C. Brenner and L. R. Scott, The mathematical theory of finite element methods, *Springer*, 2008.

[3] S. Linge and H. P. Langtangen, Programming for Computations - MATLAB/Octave, *Springer Open*, 2016.

****

14 TOPOLOGY

On convex structures in quasi-metric spaces

Mcedisi Sphiwe Zweni[a] & **Olivier Olela Otafudu**[b]
[a] North-West University, South Africa
24410373@nwu.ac.za
[b] North-West University, South Africa
olivier.olelaotafudu@nwu.ac.za

Abstract

K¨unzi and Yilzid introduced the concept of convexity structures in the sense of Takahashi in quasipseudometric spaces. In this talk, we continue the study of this theory, introducing the concept of W-convexity for real-valued pair of functions defined on an asymmetrically normed real vector space. Moreover, we show that all minimal pairs of functions defined on an asymmetrically normed real vector space equipped with a convex structure which is $W-$convex is translation-invariant.

Keywords: Convex quasi-metric, Isbell-hull, asymmetric norm, $W-$convexity.

References

[1] Takahashi, W. (1970):A convexity in metric space and nonexpansive mappings, I. *Kodai Math. Sem. Rep.* 22 142-149.

[2] K¨unzi, H.-P.A. and Yildiz, F. (2016):Convexity structures in T0-quasi-metric space. *Topology Appl.* 200 2-18.

[3] Shimizu, T. and Takahashi, W. (1992):Fixed point theorems in certain convex metric spaces. *Math. Japon.* 37 855-859.

[4] Otafudu, O. O. and Zweni, M. S.(to appear): W-convexity on the Isbell-convex hull of an asymmetrically normed real vector space. *Filomat journal*.

****

Definition of an Asymmetric inner product space

Yolanda Mutemwa Adem
University of Cape Town
Joint work with Prof. Hans-Peter Künzi and Dr Julie Condadie

Abstract

An asymmetric norm on a real vector space X is a function $p : X \to [0, \infty)$ such that

(a) $p(x + y) \leq p(x) + p(y)$ for all $x, y \in X$;

(c) $p(\lambda x) = \lambda p(x)$ for $x \in X, \lambda \geq 0$;

(c) $p(x) = 0 = p(-x) \Rightarrow x = 0$.

Such norms have applications in theoretical computer science, semigroup theory and approximation theory and have been extensively studied in recent years (see, for example, the monograph [2]).

In the classical theory of real normed spaces, norms that can be defined in terms of an inner product via the equation $||x||^2 = \langle x, x\rangle$ have been charactarised in many different ways (for this, see [1]). The best known of these is the characterisation of such a norm as one that satisfies the parallelogram identity

$$||x + y||^2 + ||x - y||^2 = 2(||x||^2 + ||y||^2).$$

For such a norm an inner product can be defined using the formula

$$\langle x, y\rangle = \tfrac{1}{4}(||x + y||^2 - ||x - y||^2).$$

In this talk we aim to define an asymmetric inner product, using the above characterisation of the (symmetric) inner product as starting point. Characterisations of the usual (symmetric) inner product as given in [1] will be used in a similar way to define possible asymmetric inner products, and the relationship between them investigated.

References

[1] D. AMIR, *Characterizations of Inner Product Spaces*, Birkhäuser, Basel, 1986.

[2] S. COBZAŞ, *Functional Analysis in Asymmetric Normed Spaces*, Birkhäuser, Basel, 2013.

[3] J. FERRER, V. GREGORY, C. ALEGRE, Quasi-uniform structures in linear lattices, *Rocky Mountain J. Math.*, **23** (1993), 877 – 884.

****

Index